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ABSTRACT

Bilingual mathematics competencies and competency-coordinated activities are provided in the four sections of this curriculum guide for kindergarten (pre-operational) and grades 1-3 (concrete-operational) children. Topic areas for kindergarten include: classification (logic); comparing/ordering/graphing; quantitative; measurement; geometry; patterning/predicting; and construction. Topic areas for grades 1-3 include: classification (logic), comparing, and ordering; numeration; notation; measurement; basic facts (addition and subtraction of whole numbers); algorithms for operations; graphing; geometry; patterns; and fractions. Each section begins with the list of competencies related to the topic area, followed by activities for learning the competency. Included with the activities are behavioral objectives, instructional strategies, a list of materials needed, and when applicable, references to text materials. Since the curriculum is intended to be a developmental/diagnostic/prescriptive curriculum, competencies and activities are not grade bound; pre-operational children, regardless of grade placement, need pre-operational activities/experiences; likewise, concrete-operational children in kindergarten need concrete activities/experiences. In addition, before each objective in the concrete-operational sections (grades 1-3) is a conservation task appropriate for that objective. Children successful with this task should continue with the objective activities while those not successful must continue with activities suggested in this area of the pre-operational (kindergarten) section. (JN)

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DYSART UNIFIED SCHOOL DISTRICT

ESEA TITLE VII
BILINGUAL PROGRAM

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Mathematics Curriculum Guide
for
Kindergarten through 3rd Grade

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The following teachers assisted in developing the goals and objectives of this curriculum and beginning pilot projects.

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To the Dysart Bilingual Teachers and users of this curriculum guide:

The following competencies and competency/coordinated activities are not intended as a completed curriculum but as the beginning of an ongoing curriculum building process. The competencies are my recordings of the goals we established for the pre-operational and the concrete operational child in mathematics during our August, 1979, meetings. The activities are my suggestions to get the curriculum process rolling. The binding is loose-leaf because it is assumed that each of you will have suggestions for learning experiences that will help students develop these competencies.

It is important to remember that these competencies and related activities are not grade level bound. The pre-operational child, regardless of grade placement, needs the activities and experiences that are coordinated with the pre-operational competencies. The child who is at the concrete operational stage should be exposed to the activities coordinated with the concrete operational competencies even if the child is in kindergarten.

Before each objective in the concrete operational curriculum is a conservation task appropriate for that objective. If the child is successful with this task continue with the objective activities. If the child is not successful s/he must continue the activities suggested in this area of the pre-operational curriculum. Children may be ready for some concrete operational areas while needing to remain in the pre-operational curriculum for other areas.

This curriculum is intended to be a developmental/diagnostic/prescriptive curriculum. This assumes that the curriculum will be adjusted to meet the needs of the individual child at his/her developmental stage. But it does not mean that each child must do each activity alone, there will almost always be a group of students in your classroom who will benefit from the same activity. Your suggestions for making such activities more useful in the classroom will be especially welcomed.

Sincerely,
Helen Cheek
Helen Cheek
Math Consultant

HC:kj

APPENDIX II

Diagnostic Developmental Activities

In order to achieve any meaningful understanding of numbers and operations, the student must possess certain prerequisite intellectual capabilities. The eight cognitive operations listed in Appendix I can be grouped into two categories: (1) those dealing with conservation (invariance) and (2) those dealing with classification.

The following activities provide a means of diagnosing whether a child has developed sufficient competence relative to each intellectual skill. The activities should be presented to each child in one-to-one interview situations. The results of the interview and the suggestions provided in Appendix I should assist you in placing each student in the proper activity level.[1]

Note that a student must deal effectively with logical classification, seriation, and conservation of number in order to define and compare numbers. Students who do not meet these criteria should be encouraged to experience many concrete and representational activities, since activities at the symbolic level will be of little benefit. Children develop competence with these three cognitive operations at about the same time—i.e., at six or seven years of age. Some children, however, achieve these intellectual capabilities a little earlier, and some much later.

Students who do not deal effectively with set inclusion are not intellectually prepared to work with the operations of addition and subtraction. Rarely do children achieve this concept before seven years of age. This implies that formal addition and subtraction instruction, especially at the symbolic level, should be started at about the second grade. First-grade students might then have a greater opportunity to experience concrete and representational activities.

Once students are dealing effectively with transitivity, instruction in multiplication can begin. Interestingly enough, understanding of transitivity is also achieved at about age seven. This suggests that instruction in addition should begin a little later and multiplication a little earlier than scheduled in the curricula utilized by most districts in the United States.

Competence with conservation of area, length, and mass should be assessed prior to addressing such concepts in measurement activities or utilizing measurement models which provide a basis for more complex mathematical concepts, such as fractions, decimals, and various aspects of geometry.

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[1] For detailed discussion see Richard W. Copeland, *How Children Learn Mathematics* (New York: McMillan, 1974).

Souviney, R. et al. Mathmatters. Goodyear, 1978.

Chapter 3

DEVELOPING MEASUREMENT CONCEPTS

Before teachers begin to design instructional activities, some thought must be given to the nature of the population for whom the activities will be developed. Using the space provided below, jot down descriptive words that complete the sentence.

People (children) are by nature. . . . . .

______________________________ ______________________________

______________________________ ______________________________

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Among the items listed, there are probably such things as curious, spontaneous, active, "doers" and independent but, at the same time, dependent. If one truly believes that these words characterize the basic nature of children, then what does this say about the types of learning activities that are designed for them? Some of our pedagogical leaders would answer that question with the following:

> Whatever we do in teaching depends upon what we think people are like. The goals we seek; the things we do; the judgments we make; even the experiments we are willing to try, are determined by our beliefs about the nature of man and his capacities.[1]

Measurement instruction is concerned with mental structures and operations with concrete objects as well as with mental images in a way that enhances and depends upon the learner's ability to think. Thinking is affected primarily by the maturation level of each learner and by his/her experiences. The assimilation and accommodation of the brain to actions and objects in the environment fosters the development of the thinking processes. The complex processes of thought are acquired gradually, developing from the simple forms found in the young infant.

What the psychologists have observed is of the greatest importance to teachers in planning the experiences they will offer to children. It is these experiences of making discoveries through his/her own actions and attempting to express what he/she has discovered which enables a child to "think metric" instead of merely carrying out operations mechanically and without real understanding.

Learning activities that are consistent with the nature of children will pose problems that challenge each child to manipulate materials to determine answers. If, in fact, children are active, the learning situations that teachers arrange will capitalize upon this characteristic. When planning measurement

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[1] *Perceiving, Behaving, Becoming: A New Focus for Education Association for Supervision and Curriculum Development*, NEA 1962 Yearbook, Washington, D. C.

14. Oklahoma State Dept. of Education, Metric Handbook.

activities, for instance, teachers will have children moving about measuring objects inside the classroom and on the playground. Measurement must be "learned by doing." Children should not be told how to measure; they should be given materials and allowed to experiment and try to solve measurement problems for themselves.

> I hear and I forget;
> I see and I remember;
> I do and I understand.
>
> *(Old Chinese Proverb)*

The belief that children are naturally curious means that teachers do not have to MAKE children learn, but rather arrange the environment to allow them to make predictions and test hypotheses.

> The study of how children come to measure is particularly interesting because the operations involved in measurement are so concrete that they have their roots in perceptual activity (visual estimates of size, etc.) and at the same time so complex that they are not fully elaborated until somewhere between the ages of eight and eleven.[2]

* Consideration must be given to the volumes of substantive research concerning the intellectual development of children compiled by the Swiss psychologist, Jean Piaget. Figure 1 presents a summation of the four stages of intellectual development as posited by Piaget.

The first stage, sensory-motor, lasts from birth to approximately one and one-half years. This stage is characterized by direct action -- actions of grasping, looking, sucking, etc. In the latter part of this stage, the child moves from spontaneous movements and reflexes to actions based on an objective or end followed by a search for an appropriate means to that end. This, Piaget characterizes as intelligence. (An act of intelligence is defined here as one in which there is first purpose or end, and then a search for appropriate means to that end.)

Beginning at one and one-half or two years to seven or eight years of age, children are in the preoperational stage. This is typically referred to as the representation or symbolic stage. There is no reversible thought process at this stage. In other words, just because a child knows that 3+2=5, does not necessarily mean that he/she will "know" that 5-2=3. Reversibility in thought comes during the next stage of intellectual development.

Concrete operations (the third stage) extends from seven or eight to eleven or twelve years of age. Most of a child's elementary school years are spent in this stage of developmental thought. This stage is characterized by logical thought processes based in part on the physical manipulation of objects. "Conservation" or "invariance" is a major construct developed during this stage. Piaget uses the term "conservation" to refer to the constancy of a given amount of substance, regardless of how it changes in shape or position. For example, a child is shown two sticks the same length lying side by side. One of the sticks is moved up or down a few centimetres and the child is then asked if the sticks are the same length. When the child understands that the length of the sticks remains the same and rejects what perception tells him/her, he/she is using logic and has arrived at the concrete operational thought level because he/she can "conserve" length.

The fourth developmental level is referred to as formal operations. This stage does not usually occur until eleven or twelve years of age.* It is in this stage that the child develops the ability to reason or hypothesize with symbols or ideas, rather than physical objects as a basis for thinking.

*or later - many college students and other adults have not reached this stage of development.

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[2] *Jean Piaget, Barbel Imhelder, and Alina Szeminska. The Child's Conception of Geometry. New York: Basic Books, Inc. 1960, p. vii.*

Oklahoma State Department of Education, Metric Handbook.

Figure I. Intellectual Developmental Stages--Adapted from the Writings of Piaget and Inholder, by Odvard Egil Dyrli

Code	Developmental stage	General age range	Characteristics of stage pertaining to problem - solving activities: comments and examples
—	Sensorimotor	Birth to approximately 18 months	--Stage is preverbal --An object "exists" only when in the perceptual field of the child --Hidden objects are located through random physical searching --Practical knowledge is developed which forms the sub-structure of later representational knowledge
A I B	Preoperational or "representational"	18 months to 7-8 years	--Stage marks the beginning of organized language and symbolic function, and as a result, thought and representation develop --The child is perceptually oriented, does not use logical thinking, and therefore cannot reason by implication --The child is simple goal-directed; activity includes crude trial and error corrections --The child lacks the ability to coordinate variables, has difficulty in realizing that an object has several properties, and is commonly satisfied with multiple and contradictory formulations --Since the concept of conservation is not yet developed the child lacks operational reversibility in thought and action
A II B	Concrete operations	7-8 years to 11-12 years	--Thinking is concrete rather than abstract, but the child can now perform elementary logical operations and make elementary groupings of classes and relations (e.g. serial ordering) --The concepts of conservation develop (first of length, next of matter, then of weight and finally of volume) --The concept of reversibility develops, yet inversion and reciprocity are not integrated into a single system --The child is unable to isolate variables and proceeds from step to step in thinking without relating each link to all others

Oklahoma State Department of Education, Metric Handbook

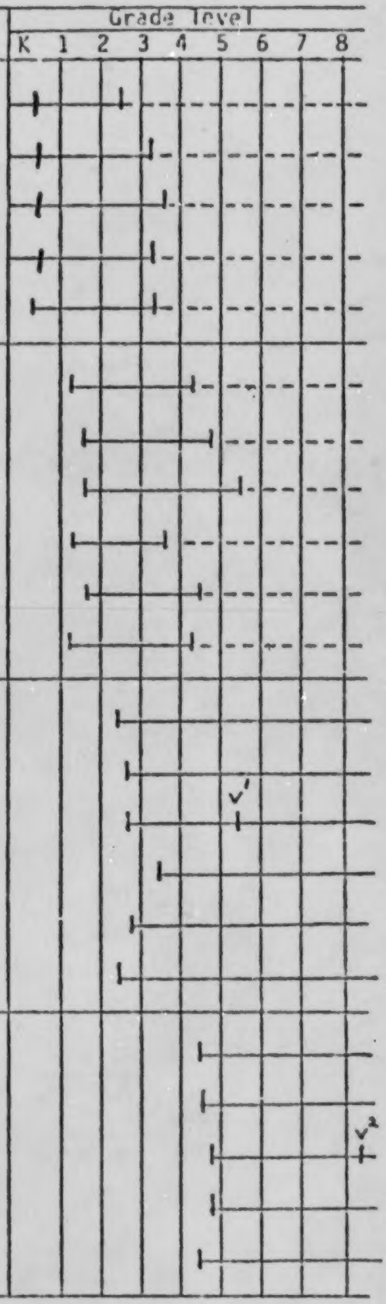
Code	Developmental stage	General age range	Characteristics of stage pertaining to problem - solving activities: comments and examples
A		11-12 years to 14-15 years	--This stage of formal (abstract) thought is marked by the appearance of hypothetical-deductive reasoning based upon the logic of all possible combinations, the development of a combinatorial system and unification of operations into a structured whole. --The INRC group of operations appear (identity, negation, reciprocal and correlative transformations comprising the INRC cognitive structue) --The development of the ability to perform controlled experimentation, setting all factors "equal" but one variable (in substage III A, the child's formal logic is superior to his experimental capacity). Individuals discover that factors can be separated by neutralization as well as by exclusion. --Reversal of direction between reality and possibility (variables are hypothesized before experimentation). Individuals discover that a particular factor can be eliminated to analyze its role, or the roles of associated factors --The individual can use interpropositional operations combining porpositions by conjunction, disjunction, negation and implication (all arise in the course of experimental manipulations).
III	Propositional		
B		14-15 years and onward	

Oklahoma State Department of Education, Metric Handbook

MODEL MEASUREMENT PROGRAM

Figure 2

State of measure/process	Quantity
Making direct comparisons between objects Language development Estimation Simple matching and comparison Ordering Simple relations and mapping Pictorial representation	Length
	Mass
	Volume*/capacity
	Time
	Area
(Comparing an object with a nonstandard unit (including parts of the body) Language development Estimation Simple matching and comparison Ordering Simple relations and mapping Pictorial representation	Length
	Mass
	Volume*/capacity
	Time
	Area
	Temperature
Comparing an object with a standard unit (SI) Language development Estimation Simple matching and comparison Ordering Simple relations and mapping Pictorial representation	Length
	Mass
	Volume*/capacity
	Time
	Area
	Temperature
Choosing measurement units for specific tasks Language development Estimation Simple matching and comparison Ordering Simple relations and mapping Pictorial representation	Length
	Mass
	Volume*/capacity
	Time
	Area



*Solid volume (v^1) and displaced volume (v^2)

Piaget's research suggests that consideration must be given to the developmental level of each child as the instructional activities are planned for him/her. The activities need to be analyzed to insure that the child has the necessary prerequisite skills. To ask a child to perform a task for which he/she does not have the intellectual tools (is not developmentally ready for) is inhumane. This means we cannot rely upon grade level hierarchies of learning since Piaget's findings also indicate that there is nothing absolute about the age ranges for a particular stage. This means, for example, that some fifth graders may still be operating on a preoperational level in some areas.

A model instructional program is presented in Figure 2 (from California State Department of Education publication, An Inservice Guide to Teaching Measurement). If instruction is to be consistent with the nature of the learner, it must progress through these four stages of development. The first stage, Making Direct Comparisons Between Objects, involves simple matching and comparison activities such as identifying five objects in the room that are taller than you, or ordering objects from the largest to the smallest. To measure the length of one object by comparing it with another marks the beginning of measurement.

The second stage, Comparing An Object With A Nonstandard Unit, includes activities in which children measure linear dimensions of familiar objects in terms of pencils, drinking straws or pieces of string.

During the third stage, Comparing An Object With A Standard Unit, students will be involved in activities which require them to find the length of the hall in metres or determine how many millilitres of milk they drink at lunch, or collect five objects that are each one decimetre long or wide.

Choosing Measurement Units for Specific Tasks is the most advanced stage of measurement activities. Tasks that ask students to find a way to measure the thickness of one sheet of paper, or determine which metric unit(s) would be used to measure such items as a flea, a watermelon, a dog, a house, a nickle, a rope, or bucket of milk are typical for this stage.

Piaget's research has proven that making statements like, "By the end of third grade, all students should be able to.." is educationally unsound. Rather, the decision concerning when to teach particular concepts should be made in relation to the individual child's developmental level as determined by the various conservation tasks developed by Piaget. Figure 2 provides general guidelines about the appropriate grade level placement of various types of measurement activities. The lines indicate the levels at which a particular activity MIGHT be successfully done by MOST children. (Broken lines indicate that the measurement activity could be used with older children to reinforce measurement skills.)

Piaget states that if systematic measurement is to be "taught," it should not be presented before the latter part of what is usually third grade. Even then, for most children, it will have to be an experimental or trial-and-error readiness-type experience. As the activities in Chapter 4 are implemented, two factors must be considered:

1. Before attempting systematic measurement, the child must be able to conserve quantity. For example, until length of an object can be conserved, linear measurement is meaningless.
2. The child must understand the concept of partitioning a whole into congruent parts. For example, in order to measure a relatively long object, one could subdivide the object into parts the same length as the measuring instrument and then count the parts.

In summary, the instructional techniques employed to help children learn to measure should be a child-centered, "hands-on" activity approach. Each learner must make the observations, do the experiments, collect and interpret the data and draw the conclusions. For the child-centered activity approach to be most successful, the classroom should be organized in such a way that it can provide actual experiences with a variety of real objects and phenomena. Each child's discoveries are his/her own and are a direct result of his/her own actions and manipulations.

TEACHING CONCEPTS: IDEAS FROM THE PAST

TEACHERS: LET'S GET BACK TO THE BASICS (1904 style):
Carne Barnett University Extension U.C. Berkeley

SUGGESTIONS TO TEACHERS

It is confidently believed that the following suggestions, if carried out in the spirit of the true teacher, will be helpful in making this book a means of better elementary instruction;

1. As far as possible, lead the child to discover the facts of numbers for himself.

2. Objects, as beans, nails, etc., are indispensable as means of inducing thought in connection with numbers.

3. First, a clear conception of the facts of numbers; then, repeated systematic drill to familiarize them.

4. "Make haste slowly." Children aquire very slowly ideas of numbers and processes.

5. A child takes delight in doing what he is able to do well.

6. Cultivate a habit of self-reliance and clear expression.

7. The child should read understandingly every question before he attemps to answer it.

8. Solutions of questions to be prepared on slate or paper, should be expressed briefly and by proper signs. Thus, question 1, page 10: 14 cents-10cents=4cents.

9. Insist upon neatness, ACCURACY, and rapidity.

10. Develop the numbers from 11 to 20, inclusive, after the method indicated in Lessons I and II. Continue the method on the multiplication table, as illustrated on page 97, by the first four sets of factors.

11. Much depends upon the eye to see results promptly, in order to secure rapidity. Besides those exercises in naming results at sight, given in the book, others should be written on the board.

12. Refer frequently to the drill exercises, using them only a few minutes at a time. Thoughtful repetition on different days is a great help to memory.

13. Children become interested in making their own problems. Let some abstract examples be assigned for the children to change to concrete problems; as, for instance, the numbers in example 1, page 47. The pupil may say, Charles has 20c., Henry has 44c., and Tom has 17c.. How much money have they?

Model Elementary Arithmetic by H.H. Belfield, Ph.D. Scott Foresman, 1904.

Bilingual Kindergarten Math Competencies
For the Pre-Operational Child

1. Classification (Logic)

   (a) Classify objects by size, color, and the four basic geometric shapes (circles, squares, rectangles, triangles).

   (b) Choose an appropriate attribute by which to classify objects.

II. Comparing/Ordering/Graphing

   (a) Compare and contrast, using vocabulary such as more and less.

   (b) Use manipulatives to reproduce a sequence, using color, shape, and size.

   (c) Participate in making a graph, using one-to-one correspondence with manipulatives.

III. Quantitative

   (a) Count from 1 to 10 using one-to-one correspondence (rational counting).

   (b) Demonstrate how two sets of objects can be joined to form a new set, with emphasis on "one more than".

   (c) Demonstrate how one set of objects can be taken apart to form two new sets, with emphasis on "one less than".

IV. Measurement

   (a) Identify and use the terms whole and half as related to cooking and sharing.

   (b) Measure common objects or spaces in the classroom using non-standard units of measure.

   (c) Use larger and smaller to compare two areas.

   (d) Use more and less to compare the capacity of two containers.

   (e) Use heavier and lighter to compare the mass of two objects.

   (f) MONEY: Recognize penny, nickel, and dime.

V. Geometry

(a) Identify shapes around us, using vocabulary such as round, square, longest, shortest.

(b) Identify basic shapes of circles, triangles, and rectangles.

VI. Patterning/Predicting

After using manipulatives to reproduce a repeating sequence, continue the pattern.

VII. Construction

Use blocks, logs, straws, and other objects in free construction.

Bilingual Kindergarten Math-Competency-Coordinated Activities For the Pre-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY — MATERIALS

I. (a) Classify objects by size, color, and the four basic geometric shapes.

Sorting Activities:

Sorting on the overhead projector

Junk Box Sorting

Ask children to sort box of items into groups (do not specify the attribute they are to use). After they have sorted the objects have them tell you how they know which group to put things into. Ask them to re-sort them in a different way.

Beads
Blocks
Buttons
Cards
Keys
Attribute-pieces
Parquetry-tiles
Stones
Cuisenaire-rods
Books

Games

Lotto games

Concentration games

Go Fish

Attribute shape game

Teacher-made Ice Cream Cone-color game

Songs

Hap Palmer records
Learning-Basic Skills Through - Music ,
Vol. I, II
AR 540

Set Language

Show children several objects such as chalk-

I. (a) cont. Classify objects by size, color, and the four basic geometric shapes.

Set Language (cont.)

board erasers and say "This is a set of erasers". Show other sets of objects and name them, leaving them in view. Show other familiar objects, such as shells, and ask "What is this?" The children may reply "a set of shells" or just "shells". In either case, agree that they are right and say, "They are shells. This is a set of shells." After many sets have been named and are still visible, ask again about each set and say, "What is the name of this set?" Then ask them to show you the set of pencils, cups, etc.

WORKJOBS I pp. 32-33, 68-71, 76-77, 102-105

ACTIVITIES FOR LEARNING THE COMPETENCY — MATERIALS

I. (b) Choose an appropriate attribute by which to classify objects.

See I. (a) - Junk Box Sorting.

Follow same strategies for sorting such everyday items as pencils, crayons, blocks, toys, etc. Follow up each time by asking how they know which object goes in which group and whether it can be regrouped a different way. Attribute pieces are very good for multiple groupings.

Beads
Blocks
Buttons
Cards
Keys
Attribute-Pieces
Pattern-Blocks
Stones
Cuisenaire-Rods

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
II. (a) Compare and contrast, using vocabulary such as more and less, most and fewest.	
Use real objects for activities: "Which group has more blocks?" "Show me the group which has fewest beans." "Walk around the table and count your steps. Who took the most steps? Who took one step less than that? Who took the fewest steps?"	Blocks Beans Cookies or crackers Bottle tops Marbles Buttons Beads Golf tees
Place objects in a container. Have children fill a similar container with more, less or an equal amount.	Beans Macaroni Marbles Blocks
Make bar graphs to indicate such things as the number of pupils who have birthdays in each month. "In which month are the most birthdays?" "In which month are the least birthdays?" "In which months are there an equal number of birthdays?"	Paper squares Blocks Tinker Toy sticks and washers Pictures Beans
SEE ALSO COMPETENCY II. (c)	
Other comparing activities:	
Show a set of objects in a linear arrangement. With other sets in random arrangement, ask pupils to find which sets have more members and which sets have fewer members than the set being used as a model.	
Ask two children who distinctly differ in height to stand. Ask the other children which child is taller and how they know.	

II. (a) cont. Compare and contrast, using vocabulary such as more and less, most and fewest.

Have pupils attempt to compare the heights of objects that are on different baselines - for example, one child standing on a chair and another standing on the floor. Ask them what they must do to be sure which is taller or shorter or if they are the same height.

WORKJOBS I pp. 98-99

Bilingual Kindergarten Math-Competency-Coordinated Activities For the Pre-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
II. (b) Use manipulatives to reproduce a sequence, using color, shape, and size.	
Use colored cubes to make color pattern. Have the pupils copy the pattern.	Colored cubes
Make a pattern with beads of different colors, sizes, and shapes. Have children reproduce the pattern or continue the sequence.	Variety of beads Pipe cleaners
In a group of two or three children working together, one child makes a pattern and the other(s) copies or continues it.	Pipe cleaners Beads Colored cubes Buttons Colored pegs Colored paper-squares
Other types of patterns for children to reproduce;	
Color and length pattern using Cuisenaire Rods.	Cuisenaire-Rods
Size and shape pattern using attribute blocks.	Attribute-blocks
Color and shape patterns using parquetry tiles or pattern blocks.	Parquetry-tiles Pattern blocks
Make cards showing patterns using beads, colored cubes, colored pegs, Cuisenaire Rods, attribute blocks, buttons, etc. Pupils can reproduce the patterns from the cards. (After they have had experience reproducing from objects themselves.)	Cards cut from Tagboard Beads Colored cubes Colored pegs Cuisenaire Rods Attribute blocks
Geoboard and pegboard patterns	Geoboards Pegboards Colored-rubber bands Colored golf-tees and pegs

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
II. (b) cont. Use manipulatives to reproduce a sequence, using color, shape, and size.	
Other ordering activities:	
Show pupil 3 objects differing in height. Locate the one of medium height. Have pupil choose one of the remaining objects compare it to the medium one, describe the comparison and place it to one side of the medium one. Have 2nd pupil compare 3rd object to the medium one, describe that comparison, and place the object on the other side of the medium one. Have the pupils describe the resulting arrangement.	Objects of various lengths that do not include color as an attribute of length.
Show children 5 objects ordered by height. Then show 5 more objects. Arrange the 1st, 3rd, and 5th of these objects in order. Ask pupils to insert the other two object so that the set is like the first one.	
Point to one end of 3 objects arranged by height and ask the children to describe how each object is different from the one it follows. Indicate the other end as a starting place and have them describe how each object is different from the one it follows.	

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
II. (c) Participate in making a graph, using one-to-one correspondence with manipulatives.	
Provide a shoe box full of colored cubic blocks, ask the pupils to name the colors of the blocks in the box. Ask them to find out which colored blocks are more numerous. Let them sort the blocks, stack them, and arrange the stacks in a side-by-side arrangement. They can tell perceptually which stack is taller, establish the one-to-one relationship across the columns, and then tell which stack has more blocks.	Colored cube blocks.
* Translate the preceding activity to pictorial form by using drawings or pictures of blocks (or 1" graph paper) colored like the real blocks. Have the children find an appropriately colored drawing for each colored block and place the drawings by color on a paper marked off in columns. Again, without naming the number in any column, they can tell which color of block is most (or least) numerous. (Might do this "because you want to tell other people - or remember - what we found out but we also want to play with the blocks".)	Graph paper Crayons or Plain paper Construction paper squares in colors to match blocks
Measure the width of a door, using the children's hands as the unit of measure. Make a graph showing how many handspans each child used to measure the door.	Paper Miniature - pictures blocks Construction paper squares.
Make graphs to compare and illustrate the children's attributes and things they are interested in. Suggestions for graphing; Eye or Hair color Pets Birthday months Number of siblings Bus riders, walkers, other transportation	

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

II. (c) cont. Participate in making a graph, using one-to-one correspondence with manipulatives.

Suggestions for graphing;

- Lunch buyers vs. sack lunchers (or home eaters)
- Left and right handedness
- Favorite TV shows
- Color of family car
- Head size
- Food length
- Wake up hours
- Weather
- Temperature
- Plant growth

Also see Open Math Book

WORKJOBS pp. 222-223

Bilingual Kindergarten Math-Competency-Coordinated Activities For the Pre-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
III. (a) Count from 1 to 10, using one-to-one correspondence	
-Set the table in the housekeeping center for the number of children playing.	Dishes Silverware
-Count out the correct number of dishes, spoons, etc. to serve the children participating in a cooking experience.	Dishes Silverware Paper towels
- Play a game that requires counting objects such as cards, spaces, markers, bottle caps, etc.	Candyland Sorting box (egg carton with beans)
- Count real objects at every opportunity. (Example: Paper doll with each child's name hung by spring clothes pin to wire. Each day the children come into the room and move their doll to the present wire. Group begins day by counting the dolls on the present wire and comparing it to the number of children present Also count the remaining dolls as the absences.)	Blocks Buttons Snacks Paint Brushes People Pop bottle tops
-Number songs and stories	Hap Palmer-records Number Books
-Games (from Mathematics Their Way) Count and Turn (page 93) The Counting Tape (page 94) The Pendulum Game (page 95)	
-Activities (from Workjobs) Pin Cushions (pages 130-131) The Safety Pin Game (pages 132-133) Easter Baskets (pages 134-135)	
-Activities pages 117-120 of Mathematics Learning in Early Childhood.	

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
III. (b) Demonstrate how two sets of objects can be joined to form a new set, with emphasis on "one more than."	
Use manipulatives to count each of two groups separately; then combine and count the new group.	Chips Blocks Bottle caps
Children string a group of red beads and one blue bead. Count the red beads, blue bead, and all beads on the string.	Red beads Blue bead String
Teacher makes a group of objects. Then children make a group of one more than the teacher's group.	Chips Bottle caps Blocks Buttons Beans
One child selects a domino. A second child selects a domino that shows one more dot on each side than the original.	Dominoes
Songs and Fingerplays	5 Little Monkeys Jumping on the Bed This Old Man The Ants Go Marching Hap Palmer record Vol. I Learnin Bas Skills Through Music The Number March

Bilingual Kindergarten Math-Competency-Coordinated Activities For the Pre-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
III. (c) Demonstrate how one set of objects can be taken apart to form two new sets, with emphasis on "one less than."	
Count the number of objects in a group of manipulatives. Take the group apart to form two new groups and count the number of objects in each group. Repeat and form two different groups.	Chips Blocks Bottle caps Buttons Keys
Children string a group of beads and count them. Then they remove one bead and count the number that are left.	Beads Strings
One child selects a domino. A second child selects a domino that shows one less dot on each side. (Dots of 5 or less, given 5 or fewer dominoes.)	Dominoes
Songs and Fingerplays	Ten in the Bed
EXPANDING THE CONCEPT:	
Give children two unequal sets and have them match objects one-to-one and find the number left over - Refer to this amount as the "difference" between the two sets.	Chips Blocks Bottle caps Buttons Keys

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (a) Identify and use the terms whole and half as related to cooking and sharing.	
In cooking, compare a whole cup of milk with a half cup of milk; a whole stick of margarine with a half stick of margarine; a whole English Muffin with a half of an English Muffin.	Measuring cup Margarine English Muffin
In sharing Compare a whole sandwich with half a sandwich Break a large cookie into two halves. Break a popscicle into two halves. Separate a peanut shell into two halves. Cut fruit, such as an apple, into halves.	Sandwiches Fruit Popscicles Cookies Peanuts in the shell
To emphasize importance of equality in "halfness" give one item that is easily divided such as a large cookie to each pair of children with the following rule: One child divides the item in half and the second child gets first choice of the "halves."	Cookies Popscicles Small cakes Bananas
Fill any container half full of sand, beans, or other small objects.	Various containe Sand Beans Macaroni

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (b) Measure common objects or spaces in the room, using non-standard units of measure.	
Measure the width of a door, using a child's hand as the unit of measure. See Also: Competency II. (c) - Graphs	
Trace around each child's body. Cut out the body pictures and compare who is tallest and shortest.	Paper, Pencil Scissors
Cut a strip of paper that matches the child's height. Trace around the child's hand to measure how many handprints tall the child is. Repeat by tracing around the child's food. How many pupils are at least "six feet" tall?	Paper, Pencil, (Adding Machine Tape works especially well here)
Measure a table in the room, using a variety of non-standard units of measure such as a book, a shoe, a spoon, a large block, or a pencil.	Book Shoe Spoon Large block Pencil
Make a finger graph (See Open Math Resource Book)	Paper Pencil or felt-tip pen
Ordering activities at bottom of II.(b) should be experienced first.	

Bilingual Kindergarten Math-Competency-Coordinated Activities
For the Pre-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (c) Use "larger" and "smaller" to compare two areas.	
Matching by size	Playing cards 3x5 index cards 5x8 index cards Blocks of various size
Creating designs of varying sizes	Geoboards and rubber bands Pegboards and golf tees
Comparing areas:	
Sides of boxes	Various size boxes
Books	Various size books
Sheets of paper	Various size paper
Footprints and handprints traced on paper	Paper & pencil
Flannelboard objects	Flannelboard Various felt pieces
Use a large square or other figure as a base, and see how many smaller shapes it takes to cover it.	Tiles Index cards Flannelboard shapes Parquetry blocks & cards Tangrams
Find largest and/or smallest object in a box or bag by feel.	Magic Box Magic Sack

IV. (d) Use more and less to compare the capacity of two containers.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
Compare the capacities of different containers: How many juice cans of water does it take to fill a larger container? How many shovels of sand does it take to fill the sand pail? How many blocks does it take to fill a shoebox? - an empty milkcarton?	Sand table Water talbe Measuring cups Various containe Rice Beans Blocks
Provide liquids and other "pourables" such as rice, sand, lentils, or beans, and a variety of containers, some identical in shape and size, some varying in shape but the same in capacity, and some differing in capacity. Let children pour from one container to another. They can see that containers of quite different dimensions can hold the same amount.	Containers Rice Sand Lentils Beans Colored water
(Careful attention should be given, in guided activities, to the acquisition of the language needed to describe the result of pouring the contents of one container into another.)	
Have the children compare the capacities of two containers: "This holds more than (less than, almost as much as) that." Have them order three or more containers by capacity. Let them label the ones that hold the most and the least.	Containers Water Sand Rice
Put sand into a transparent container. Have children fill similar containers with more, less, or equal amounts of sand.	Sand table Transparent-container (similar in size and shape

Bilingual Kindergarten Math-Competency-Coordinated Activities
For the Pre-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (d) cont. Use more and less to compare the capacity of two containers.	
Cooking experiences:	
Compare the change in bulk of various foods due to cooking.	Cooking equipment Apples Cake mix
Compare the capacities of various measuring spoons and cups.	
Snack-time experiences:	
Compare capacities of various serving containers.	Pitchers glasses Pudding cups

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (e) Use heavier and lighter to compare the mass of two objects.	
Compare the weights of various objects by feel.	Bricks Stones Blocks Feathers Various fruits Books Popcorn balls
Children use a balance scale to compare weights.* Do equal numbers of similar objects balance? What happens when different numbers of similar objects are compared. How does the balance scale tell that one object is heavier than another?	Balance scale Crayons Beans Beads Blocks Stones
Children use a metric scale to compare their weights. What do the numbers on the scale mean? Who is the heaviest? Lightest?	Metric scale
* INTRODUCING BALANCE SCALE:	
Have the children compare the weight of two objects of grossly different weights by lifting. Establish which is heavier and which is lighter and have them predict what would happen to the balance scale if one were placed in each pan. Have them check their predictions. When the children agree that the heavier object causes the arm from which it is suspended to come down, let them compare several different pairs of objects, this time reading the balance beam	Balance scale Various objects

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (e) cont. Use heavier and lighter to compare the mass of two objects.	
(scale) rather than relying on their physical kinesthetic sensations to say that one object is "heavier than", "lighter than," or almost as heavy as" another object.	
Provide a balance scale and from two to six look-alike clay balls, with one ball heavier than the others (a metal ball can be placed inside). Have the children use the balance to find the heavier ball.	Balance scale Six clay balls One metal ball

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (f) MONEY: Recognize penny, nickel, and dime.	
Grocery store simulation Make purchases with play money	Play money Cash register Items to buy
Children use play money to pay specified amount for special treats.	Coin stamps to make money (DLM) Special treats
Games:	
Match number of pennies to nickel and a dime	Lotto Money-Game, Development Learning Materials
Match and name coins	Concentration game (made from coin stamps or pictures)
WORKJOBS I pp. 224-227	

Bilingual Kindergarten Math-Competency-Coordinated Activities For the Pre-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
V. (a) Identify shapes around us, using vocabulary such as round, square, longest, shortest.	
Use real objects for activities: "Show me the longest string." "Who has the longest foot?" "Pick out the biggest block."	String Blocks Keys Beads
Shape Pictures On flannelboard cut from old magazines and pasted on posterboard	Flannelboard felt pieces magazines catalogs Paste Posterboard
Making Shapes	Geoboards Blocks Attribute blks Tinker Toys Pipe cleaners
Games:	
Fruit Basket Upset (using shapes)	
Matching	Dominoes Cards Lotto
Finding shapes in the environmnet	
Songs	Hap Palmer records Vol. II Learning Basic Skills through Music Math Readiness AR 540
Books:	Round is a Pancake
Filmstrips and Movies	Bomar Artworld Series Things that are Round

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
V. (b) Identify basic shapes of circles, triangles, and rectangles.	
Gather a collection of objects, such as checkers, buttons, bottle caps, small wheels, sheets of paper, corks, crayon boxes, stamps, pennants, toy irons, cards or sheets of paper cut diagonally, or other objects that can be classified by an attribute of shape. Select one object, such as a cork, and place it on one table and select another object with distinctly different shape, such as the crayon box, and place it on another table. Place a stamp with the crayon box and a checker with the cork. Continue to select objects or ask children to select objects, one at a time, and have the children indicate the table on which the object should be placed. Verify that their suggestion is correct, or if it is not, say "No, let's put the ___ on the other table." After all the objects have been classified into one or the other of the two sets, ask what is alike about the objects in each set. Check each suggestion to see if it applies to all the objects in that set. Although you have imposed a means of classification, the children may not perceive it. Try to look through their eyes, not yours.	buttons bottle caps pennants checkers corks crayon boxes stamp paper cards
Blindfold a child and have him trace his fingers around the outline of the cutout (circular, rectangular, or triangular) and then give the cutout to you to conceal before taking off the blindfold. Have him rely on his kinesthetic memory to trace the shape in the air with his eyes open and describe it as being round or as having sides and corners.	cutouts of basic shapes blindfold

TRY TO AVOID THE FOLLOWING INCORRECT CONCLUSIONS BY CHILDREN:

That a circle can be held in the hand;

That a "shape" is associated with a particular color;

V. (b) cont. Identify basic shapes of circles, triangles, and rectangles.

That a "shape" is made of posterboard (or felt, etc.);

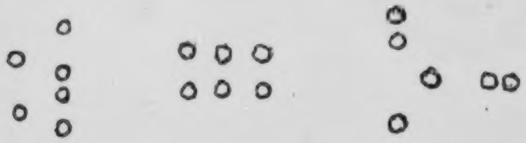
That "square" and "rectangle" are mutually exclusive categories;

That triangles have at least two equivalent sides;

That CIRCLE, RECTANGLE, AND TRIANGLE are three deminsional shapes.

To prevent such incorrect conclusions, activities with objects should be planned to develop a general concept of shape, to give the children the opportunity to observe and describe likenesses and differences in the shapes of objects, and, finally, to enable the children to learn the names for the most common shapes.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
VI. (a) After using manipulatives to reproduce a repeating sequence, continue the pattern.	
See Competency II. (b)	
Using blocks, shells, dry pasta, or anything of distinctive shape, color, or other attribute, begin a pattern (eg. 2 red blocks followed by 2 green blocks). Have the children make a pattern directly below the one shown and just like it, and another directly below that. They have replicated the pattern.	blocks shells dry pasta attribute pieces beads
Show another pattern, such as two oyster shells and one snail shell. Have the children repeat that pattern in a linear arrangement following the original. They have continued, or extended, the pattern.	various shells
Show another arrangement (eg. 2 large blocks, 2 small blocks, 2 large blocks). Have a child choose the next block and the next one. Ask why each of these things comes next. (NOTE: What she chooses next depends on the pattern she sees.)	various sized blocks
Show another pattern (a piece of boknot pasta, a piece of elbow macaroni, a piece of rigatoni, a piece of bowknot pasta, a piece of elbow macaroni). Ask the child to show or tell what the pattern is and two choose the next item in the pattern.	various kinds of dry pasta
Show a linear pattern of red and blue squares of construction paper glued to a sheet of paper. Cover one of the cutouts with a strip of white paper. Ask, "What is covered? How do you know?" Cover more than one of the cutouts (construction paper squares) and ask, "What color are the ones that are covered? How do you know?"	Construction paper (2 colors) Scissors Paste Paper

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
VI. (a) cont. After using manipulatives to reproduce a repeating sequence, continue the pattern.	
NON LINEAR PATTERNS	
Show the child a row of evenly spaced dots and have her put a counter on each of the dots. Ask her to take these counters and make a pattern or arrangement. Then have her match another set of counters to the dots so that she now has another set with just as many counters in it. Say, "Try to make these look different." Here are three possible patterns using six counters. 	checkers or other similar items
Show the child different arrangements of six or more blocks. Ask how the different arrangements are alike or different. Challenge the child to build a different arrangement with the same number of blocks.	Blocks
Begin making an arrangement on a geoboard with differently shaped colored beads or spools. "Which bead comes next? Where do we put it? How do you know?" Sequence can involve spatial arrangement or only color and shape.	Geoboards Beads Spools

Bilingual First Grade Math Competencies for the Concrete-Operational Child

I. Classification (Logic)/Comparing/Ordering.

"The student will have experiences classifying, comparing, ordering; the use of number and reversability allows the student to extend these activities through numerical descriptions."

(a) Classify objects by size, color, and the four basic geometric shapes (circles, squares, rectangles, triangles) and classify sets by the number of members.

(b) Compare sets of objects and verbalize about their size.

II. Numeration

"The student will have experiences with numeration; extensive use of numeration models will be used with many grouping and trading activities for developing the place value concept."

(a) Group bundles of ten popscicle sticks or ice cream spoons.

(b) Play games that involve bundling and unbundling sticks.

(c) Count orally:

by ones to 50
by twos to 20
by fives to 50
by tens to 100

(d) Use the concept of zero in addition and subtraction.

III. Notation

"The child will learn the written symbols associated with numerals, operations, and measurements after having concrete experiences with the concept and developing an oral label for the concept.

(a) The child will read and write the numerals from 0 to 9.

(b) The child will read and write the numerals from 10 to 100.

(c) The child will read and write the symbols for "greater than" "less than" and "equal to."

(d) The child will read and write the equations for the basic facts with sums to 10 in addition and subtraction (numerals, operation signs, equal sign).

(e) The child will begin writing number sentences expressing inequalities with numbers of nine and less (numerals, operation signs, greater than and less than signs).

(f) The child will read and write the signs for dollars and cents (both decimal notation and separate ¢ notation).

IV. Measurement

"The student will have experiences with measurement; early experiences with non-standard measurement will be followed by many activities at every level using both the metric and English systems for length, mass, capacity, area, and temperature. The measurement of time will include sequencing, and many activities of short (hours, minutes, seconds) and long (day, week, month, seasons, year) duration.

(a) Measure objects, using ruler, yardstick, meterstick, and tape measure.

(b) Illustrate areas of rectangles on a geoboard.

(c) Measure water into cups, pints, quarts, and liters.

(d) Use balance scale to find approximate weight of common objects.

(e) Read time to the nearest hour.

(f) MONEY: Identify pennies, nickels, dimes, quarters: Give their equivalent amounts in pennies.

V. BASIC Facts (Addition and Subtraction of Whole Numbers)

"The student will have experiences with basic facts; once a child is able to demonstrate an understanding of an operation through the use of concrete materials, drill activities for memorizing the basic facts will be implemented."

(a) Use real objects to add and subtract numbers with sums to 18.

(b) Recall addition facts through sums of 10.

(c) Recall subtraction facts through sums of 10.

VI. Algorithms for Operations

"The student will have experiences with algorithms for operations; introduction for multidigit numerals via concrete materials and to be taught as a way of recording the concrete experience."

See competencies II.(a) and (b) and competency IV. (a) for introductory experiences. Algorithms for multidigit numerals will not be introduced at first grade.

VII. Graphing

"The student will have experiences organizing and presenting data; quantities will be graphically represented in numerical form as well as pictorial."

(a) Make picture graph with information student helped to gather.

(b) Make bar graph with information student helped to gather.

VIII. Geometry

"The student will have experiences with shapes and space - extensive work with geoboards in exploring measurement of perimeter, area and angles."

See competency IV. (b)

(a) Identify basic shapes of circle, triangles, rectangles, and squares.

(b) Construct basic shapes on geoboards.

IX. Patterns

"The student will have experiences with patterns; patterning will be continued with concrete materials (eg. attribute blocks) but will now include numerical patterns as well."

(a) Put objects into a sequential pattern.

(b) Describe the pattern created with objects (include quantitative terms).

(c) Complete number patterns of basic counting by ones, twos, fives, and tens when shown a sequence with one or more numeral missing.

X. Fractions

"The student will have experiences with fractions; early concrete experiences will develop the language of fractions (including physically adding fractional parts with the same name and naming the new quantity). Numeration (notation) will be introduced once the child can divide."

(a) Identify one-half, one-third, and one-fourth of such things as a cup of water and of the basic geometric shapes.

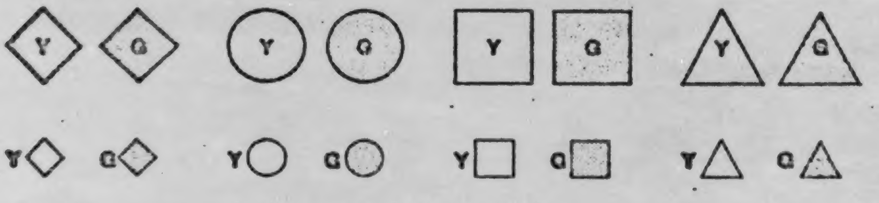
(b) Handle and count fracitonal models (eg. egg cartons or fraction bars) to amounts larger than one whole (eg. five thirds).

Logical Classification

(Number Readiness)

•*Purpose:* This activity evaluates a student's ability to sort (classify) objects consistently based on observable likenesses and differences.

•*Materials:* A set of shapes similar to those below. Each shape should be provided in two colors.



•*Procedure:* Have the student dump the blocks onto the table and ask him to sort them into two groups so that each group is alike in some way. Children who do not classify logically will have difficulty with this problem. They may change their grouping criteria in the middle of the problem or may simply make pretty patterns with the shapes. In most cases, children between the ages of two and six fall into this category. Around age six, children generally become more adept at solving this problem but nevertheless require some help with organization in order to ensure that all the blocks will fit into one group or the other. Between the ages of six and eight, most children are able to solve this problem independently and without hesitation, providing complete descriptions of their grouping criteria.

Until a child can classify logically, it is not advisable to introduce number concepts at the abstract level. Number concepts should remain implicit in the concrete materials and not formally defined by means of symbols.

Souviney, R. et al. Mathmatters. Goodyear, 1978.

Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
I. (a) Classify objects by size, color, and the four basic geometric shapes (circles, squares, rectangles, triangles) and classify sets by the number of members.	
See pre-operational activities I. (a) for additional ideas.	
Give children a set of attribute blocks and tell them to sort the blocks into two groups (give them two boxes) so that every thing in each box is related. After they have finished ask, "How are these alike?" "Can you sort them into two boxes again but in a different way?" (Continue this until they have sorted by all double attributes such as large and small, thick and thin, round and not round.) Next give them the number of boxes that are equal to the number of colors and shapes in the set and ask them to continue grouping. Always ask how the groups are chosen, "How are these alike?" (If attribute blocks are not available assemble a collection of cars, pencils, and blocks - some red, some blue, some yellow for instance, and some larger than others. Also, attribute pieces may be made from colored construction paper and laminated, however, they will lack the attribute of thick and thin.)	Attribute pieces
Bring out a variety of objects. Begin to put the objects into two sets, perhaps on a piece of tagboard marked into two sections. Into one set put things with a common property or identity - animals, perhaps - and into the other set objects that are not animals. As soon as they think they know it, have the children tell the name of either set. They will easily name the set of animals; the difficulty comes in finding a name for the other set. They may suggest names like "things" or "toys" or "objects",	Numerous small objects

Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
I. (a) cont. Classify objects by size, color, and the four basic geometric shapes (circles, squares, rectangles, triangles) and classify sets by the number of members.	
but if any of these were the name of the set then all the animals would be members of that set, too, and they are not. If no satisfactory name can be suggested, put that display aside, still in view, and begin another partitioning, such as "yellow" and "not yellow". Again seek names for the two sets. If several such examples do not elicit the desired response from the children, return to the set of animals and say,"Listen while I think how I'll decide where this one goes. Is it an animal? Yes; so I'll put it here with the set of animals. Is this an animal? No; so I'll put it with that set. This is the set of animals. What could be a name for that set?"	
Display sets of objects whose only common attribute is the number of members (eg. 5 gold stars, 5 red balls, 5 blue rectangular regions, 5 pink erasers, 5 yellow pencils, etc.) Ask the children to tell you what all of the sets have in common. (If all other attributes have been controlled, you should have no trouble eliciting the attribute of number from a group of children, however one child alone may not be able to recognize number as an attribute.)	Sets of small objects (eg. gold stars red balls blue rectangular regions yellow pencils silver jakcs, etc.
WORKJOBS I pp. 144-145	

Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
I. (b) Compare sets of objects and verbalize about their size.	
In a small group setting, give the children a box of pencils (or similar objects) and ask whether there are more red pencils or yellow pencils. How many more? Are there enough pencils for each child in the group to have one? Could each boy get a red pencil and each girl get a yellow pencil? Such questions focus on the relative sizes of sets of objects and encourages children to verbalize about them. This activity can assist the teacher to identify the students who can merely repeat the names of numbers that they have memorized in rote fashion with no true understanding of their relative size and order.	Box of pencils (or other objects)
Have the children sort sets of objects so that in one location are sets with, for example, eight members, in another location are sets with fewer than eight members, and in a third location are sets with more than eight members. Ask the children to explain to other children what the rules for sorting the sets are in their own words.	numerous sets of object (eg. blocks buttons dolls dishes books toys)
Show the children a set and have them find another set that has one (two or three) member (s) more. "How do you know it has one member more? Show me."	numerous sets of object
Display pictures and have the children put together those pictures that show sets with the same number. "How did you know these went together? Show me. Show (name of another child) how to do this activity."	pictures of sets (some of the same number of members and others different

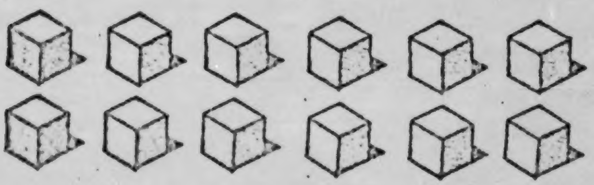
Conservation of Number

(Number Readiness)

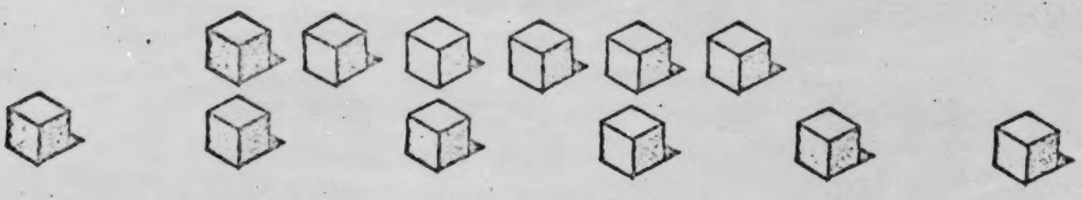
Purpose: This activity evaluates a student's understanding of the invariance (conservation) of the number of objects in a set when the elements are moved about.

Materials: Twelve identical wooden cubes.

Procedure: Make a row of six cubes in front of you. Give the rest of the cubes to the child and ask him if he has the same number of cubes as you have. Have him prove it by putting the blocks in one-to-one correspondence, as below.



Now spread his blocks out and ask him whether you both still have the same amount.



Young children (four to five years of age) have difficulty determining whether the numbers of blocks are equal and generally cannot place the two sets in one-to-one correspondence in the first place. Even if they can initially determine equivilence by putting the two sets into one-to-one correspondence, when the rows are transformed their decisions are ruled by perceptual considerations; therefore, they believe that the spread-out row contains more cubes. Over the next two years, they will become capable of putting the blocks in one-to-one correspondence, but their decisions will still be ruled by perceptual appearances. By the age of seven, children are usually able to solve this problem correctly. We then say that the child conserves number.

As with classification and seriation, it is not recommended that children be introduced to number concepts at the abstract level until they conserve number. Once children are dealing effectively with classification, seriation, and conservation of number, they are ready to be introduced to the abstract notion of number and the associated symbols.

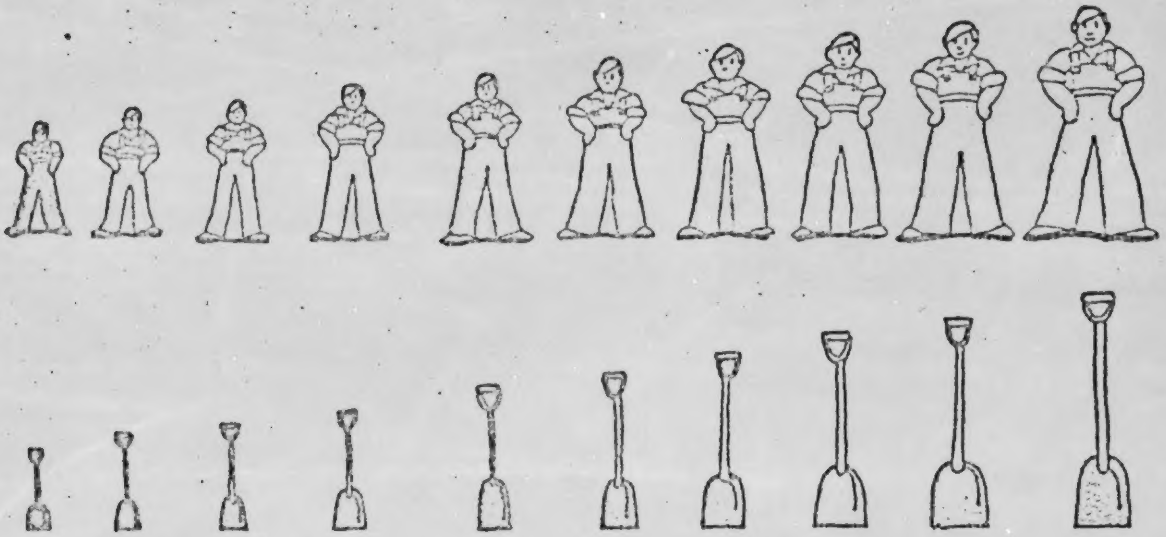
Souviney, R. et al. Mathmatters. Goodyear, 1978.

Seriation

(Number Readiness)

•*Purpose:* This activity evaluates a student's ability to place objects in serial order based on some attribute of size.

•*Materials:* Ten paper cutouts of workers gradually increasing in size, with the largest twice as big as the smallest. In addition, you will need ten paper cutouts of shovels gradually increasing in size, with the largest three times as big as the smallest.



•*Procedure:* Ask the child to arrange the workers in order from small to large. Then have him position the shovels so that each worker gets the right-size shovel. Mix the workers up and ask the child to rearrange the shovels accordingly. Next give him the shovels, point to one of the workers, and ask the child to produce the appropriate shovel.

Young children (two to five years old) are unable to seriate either the workers or the shovels. Over the next two years, most children develop the ability to solve the three seriation problems, primarily by trial-and-error methods. Generally abilities necessary to solve the three problems are acquired in the order in which the problems have been listed above. After age six, most children readily solve all three seriation problems without using trial and error.

Until a child can deal effectively with concrete seriation problems, number concepts should not be introduced at the abstract (symbolic) level. Seriation is very closely tied to the concepts of cardinal and ordinal numbers and should therefore be prerequisite to the formal introduction of these concepts.

Souviney, R. et al. Mathmatters. Goodyear, 1978.

ACTIVITIES FOR LEARNING THE COMPETENCY — MATERIALS

II. (a) Group bundles of ten popscicle sticks or ice cream spoons.

Give the pupil an odd number of counting sticks (eg. 34). "Please sort these into groups of ten and put a rubberband around each set of ten."
"How many groups do you have? How many sticks were left over?" Record the child's answer on the chalkboard or on paper as such:

bundles	left over
3	4

After several days of such recording, change the words above the —|— to read tens and ones with a discussion illustrating that the bundles are all bundles of tens and the sticks left over are counted by ones.
After a few days of the "tens" and "ones" label, reduce the labels to "T" and "O" with the explanation that "We all know that "T" stands for ten and "O" stands for "One".
After a few more days do not write any lables above the bar, "We all know the first side is for the tens and the second side is for the ones." Eventually you will be able to record the amounts without the —|— as 34 which you may still refer to as three tens and four left over.

MATERIALS:

Counting sticks
rubber bands
chalk board and
chalk or
paper and crayon

Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY — MATERIALS

II. (b) Play games that involve bundling and unbundling of sticks.

Bankers game (2 to 6 players)

Materials: 600+ (or more) counting sticks (less for fewer players)
rubber bands
one die*

Ojbect: Be the first player to have 100 sticks bundled in ten bundles of 10 sticks each.

To Play: Youngest player rolls die first, followed by each player in turn. The player rolling the largest number begins the play. After that play continues clockwise around the circle (if more than 2 play). OPTION: One player may be named banker who does not participate in the play. The banker's role is to count out the sticks and hand them to the player and to give rubber bands as they are needed.

Each player in turn rolls the die and takes from the can of sticks the number of sticks to match the number showing on the die. Each time a player acquires ten sticks, she/he must put a rubber band around the group of ten.

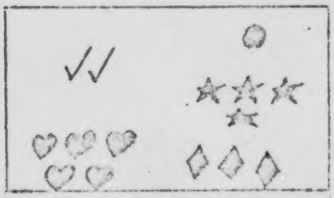
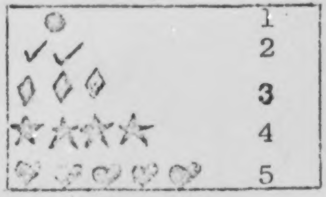
CHALLENGE: If a player is suspected of putting a rubber band around less than ten sticks he may be challenged by another player. If the challenger is correct (if there are less than 10 sticks in the challanged players bundle) the player who chal-

MATERIALS:

counting sticks
rubber bands
die

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
II. (b) cont. Play games that involve bundling and unbundling of sticks.	
lenged gets one extra roll of the die. If the challenger was wrong (if there are 10 sticks in the bundle) the player who was challenged gets one extra roll of the die.	
*A sponge rubber cube with the numerals written on it with felt tip pen is quieter than commercial dice. Also, with the numerals 4 to 9 used (in place of the usual 1 to 6) the game is speeded up.	
Gambler's game (2 to 6 players) This game is played similarly to the Bankers game except that each player begins with one large bundle of 100 sticks (each 10 sticks within the bundle should also be bundled) and the object of the game is "go broke" (i.e.; run out of sticks first). On each play the players give the bank the number of sticks to match the number the roll on the die, unbundling their bundles as necessary.	counting sticks container for sticks rubber bands die

Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
II. (c) Count orally be ones to 50, by twos to 20, by fives to 50, by tens to 100.	
Display in random order sets with one, two, three, four, and five members, as illustrated below. Have the children begin with the set of one and order the sets so that each set has one more member than the one it follows. Have the children tell the number of each set. Have them write the numerals of each set. (See second illustration.) Ask the children to look at the display and tell how many are in the set having one member more than the set of three. Continue with other questions that call attention to the "one more than" and "one less than" relationships between two adjoining sets.	numerous small objects (eg. beans buttons jacks gold stars blocks marbles rubber bands paper clips pencils, etc.)
A  B 	
BELL BAGS (see Open Math Book for pattern)	Bell bags blindfold
Have the children find how many senses they can use to order the bell bags. (eg. blindfold the child and have them use their sense of feel and hearing only)	
Count real objects at every opportunity (especially themselves and items related to themselves).	
Use bean sticks (tongue depressors with various numbers of beans glued to each stick) representing twos, fives, and tens as the basis of skip counting. (Count the beans by counting the sticks, using the total each time: 2,4,6,....)	Beansticks

II. (c) cont. Count orally by ones to 50, by twos to 20, by fives to 50, by tens to 100.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
Use the longs from multibase blocks to count in twos, fives, tens.	
Have pupils line up in pairs (for example to go to music) and then count the total by pairs (2,4,6 . . .)	
Have pupils count the fingers on the hands of a group of children by fives. (Point to each hand as the count out the total fingers pointed to so far, 5,10,15 ...)	
Buzz Game for numbers 5 and 10. (See games section)	
Use 20 white cuisenaire rods and ten rods in a display where each red is matched with two whites. Have the children touch each red rod as they count (2,4,6...)	Cuisenaire rods
Place 50 items in a box and ask the child to count them for you.	50 small items
Make playing cards with the digits 5,10, 15 ...50. Shuffle the cards. Have the pupil sort them from smallest to largest.	Blank playing cards felt marker
Count money by pennies, nickels, and dimes.	Play money

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
II. (d) Use the concept of zero in addition and subtraction.	
Creative drama: Have children act out story problems (which you or they make up) involving the addition and subtraction of zero. For example: John has 5 marbles (have John go to the front of the room and show his five marbles) and Robert has no (0) marbles (Robert goes forward adn shows his empty hands). How many marbles do these two boys have all together? Or, Helen has 7 jacks. She gives zero jacks to Maria. How many jacks does Helen have left?	Small objects such as marbles jacks pencils books crayons blocks etc.
Pill bottle addition: Use buttons or beads and pill bottles to create a hands on manipulative for addition and subtraction problems. On the top of the pill bottle write the addition or subtraction fact that is to be found in the bottle, for example, 2 + 3 and 3 + 2. In the bottle have a string with three buttons or beads and another with two buttons or beads. The two strings can be fastened together with a third string. The same beads or buttons would be in the container marked 5 - 2 and 5 - 3. For basic facts involving zero the first string will contain beads and will be fastened to an empty loop of string.	Pill bottles (plastic) string buttons or beads Felt tip pen

ACTIVITIES FOR LEARNING THE COMPETENCY — MATERIALS

III. (a) The child will read and write the numerals from 0 to 9.

See II. (c) activities

Activities:

Make 5" x 8" cards for each number from 0 to 9 with the word name for the number on one side, the numeral on the other side, and a pictorial representation under the numeral (eg. seven dots under the numeral 7). Attach the number of clothespins for the number at the top of each card.

Have children remove clothespins from several cards, mix pins, then pull out the correct number of pins to put back on the cards.

Have students copy the numeral and pictorial representation from the card.

5 x 8 cards
felt mark pen
clothespins

Use the black line masters from Mathematics Their Way to practice writing the numerals.

Remember: most numerals face the opposite direction of most letters (alphabet) and children will often reverse the numerals. Make them aware of the problem but accept it as a normal problem of maturation.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS

III. (b) The child will write the numerals from 10 to 100.

DO NOT HAVE CHILDREN WRITE NUMERALS UNTIL THEY CAN COUNT OUT THE NUMBER IN OBJECTS.

Activities:

Once children can write the digits from 0 through 9, the writing of multi-digit numerals is primarily a matter of spacing. Have children turn their lined papers side ways, make one digit in each space, and leave one or more spaces between numerals.

Graph paper is also helpful in helping pupils learn to properly space two digit numerals.

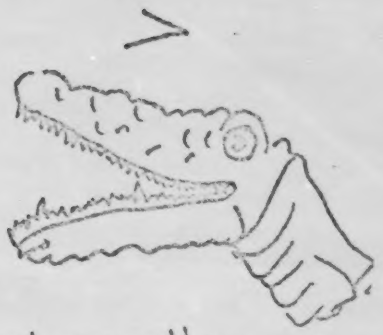
Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

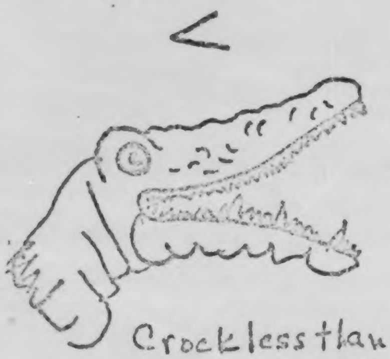
III. (c) The child will read and write the symbols for "greater than" and "less than."

See Mathmatters pp. 36-40.

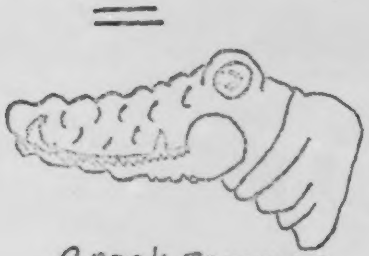
Use a puppet of the greedy duck or greedy crocodile to introduce they symbols. Then use cut out of tagboard or painted pellon to follow up showing the symbols to be a picture of the greedy animal's mouth.



Crock more than



Crock less than



Crock same as

from Mathmatters, p 37.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
III. (d) The child will read and write the equations for the basic facts with sums to 10 in addition and subtraction (numerals, operation signs, equal sign).	
Introduce the signs for plus and minus after children have used concrete objects for developing these concepts (see activities for objectives in IV). Be sure to call them the PLUS and MINUS signs. Use 1" tiles with numerals, equality and operations signs printed on with permanent felt mark pen. Have children place tiles in number sentence (equation) forms. Use dittos from Mathematics Their Way using hands as well as plus or minus sign as pictorial reminder. WORKJOBS 150-155, 164-165, 168-175,194-211.	

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

III. (e) The child will begin writing number sentences expressing inequalities with numbers of nine and less (numerals, operation signs, greater than and less than signs).

See activities for III. (d) - add the greater than and less than signs to the set of 1" tiles and have children make sentences expressing such number iteas as "4 - 1 $<$ 5.

(In the above example, students should feel comfortable using manipulatives such as Cuisenaire rods to prove their expressions.)

See Mathmatters pp. 36-40

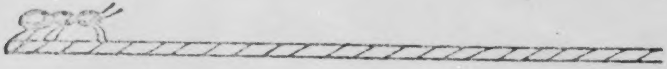
See Marcy Cook Count-A-Tile from SET A

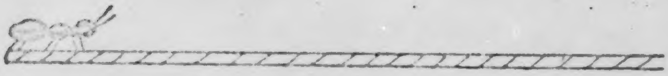
Conservation of Length

•*Purpose:* This activity will help the teacher evaluate a student's understanding of the invariance (conservation) of the linear qualities of concrete objects when they are moved about.

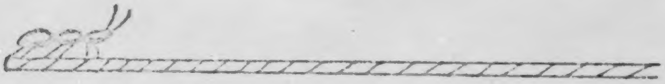
•*Materials:* Two equally long pieces of heavy cord.

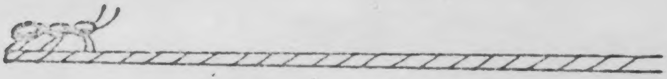
•*Procedure:* Present the following situation to the child: If two equally fast crawly-bugs were put on the end of these strings



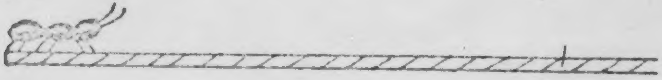


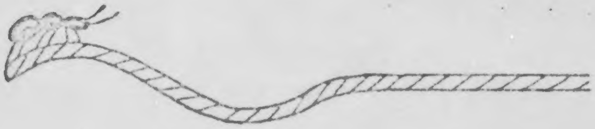
and they both went the same speed, would they reach the end at the same time? How about these two strings? (Use the same strings.)





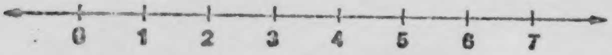
And these? (Again, use the same strings.)





Most children under six years old will not realize that the race would be a tie in the second and third cases even though they know you are using the same string. Around the age of six or seven, most children conserve length and give the correct response because they are able to reverse the process, mentally moving the strings into their original positions.

Children who do not conserve length should not be expected to measure with a ruler, since when they move the ruler it seems to get longer! The number line is of no help to a nonconserver of length. That child perceives the distance between four and five on the number line as being greater than the distance between one and two. Other, more perceptually sound models should be used in such cases.



Souviney, R. *et al*. *Mathmatters*. Goodyear, 1978.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (a) Measure objects, using ruler, yardstick, meterstick, and tape measure.	
See Preoperational activities for non-standard measures to be used as an introduction as to why standard measures are needed.	
Finding a rope for Pero: Arrange the pieces so that Pero can't get to his food. Make up a reason why the dish cannot be moved. Have the food and flower beds on opposite sides of the dog house with the food several inches or a foot closer to the doghouse than the flowers are. Tell the children they need to find a rope long enough for Pero to reach his food but not long enough for him to reach the flower bed because he always tears them up trying to bury his bones. Have the ball of twine fastened to something on the opposite side of the room. Tell them they must go to "the store and buy just the right amount of rope" for Pero. Let the children talk about how to find the right amount. If no one brings up the idea of using a third object (such as an arm, or a measuring devise) to find out how much twine is needed you may have to bring it up. Have them use some standard rule (English or metric) to mark the amount of twine needed and then go "to the store" and see if they can get the right amount.	Large cut outs for bulletins large dog doghouse dish of food flowers twine Or display with a toy dog.
Using Cuisenaire rods for standard measure: Play a guessing game with a group of children where one person (usually the teacher) holds up an item which can be measured linearly (eg. pencils, book edge, chalkboard eraser) and the children and teacher all take a guess at how many white and orange cuisenaire rods long the item is. Have every one write down their guess. Then let one of the children place the rods on or	Objects to be measured Cuisenaire rods

ACTIVITIES FOR LEARNING THE COMPETENCY | MATERIALS

IV. (a) cont. Measure objects, using ruler, yardstick, meterstick, and tape measure.

beside the object to measure it. Have them consider the orange rod as ten so that if they think there are one orange and 3 white rods they can write 13 as their guess. (Because the rods are metrically scaled, this also helps the children to internalize the metric system. However, at this point do not discuss centimeters and decimeters - just white and orange rods.)

Centers:

Try to keep a measuring center going in the classroom where children can have the fun of measuring everyday objects including themselves and their classmates. Plain meter sticks (nothing smaller calibrated) are good for measuring the large items in the classroom such as walls, chalkboards, bookcases, etc. Tape measures should be availalbe for measuring body parts.

Meter sticks (no calibration)
cloth tapes - metric
Yardstick
rulers - 12" and 30 cm

OPEN MATH RESOURCE BOOK: pp. 14, 106-108, 229, 232-233

WORKJOBS: 218-221

PRIMARY MATH LAB: 26-30

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (b) Illustrate areas of rectangles on a geoboard.	
Allow the children to make designs on the geoboards. Then ask if anyone has a rectangle in their design. Ask who has the largest rectangle. Establish the square unit between four nails as the unit of measure. BE SURE TO CALL IT SQUARE UNITS. How many square units does the largest rectangle contain? How many does the smallest rectangle contain? (Note: Some of the children will not realize that a square is a rectangle and will believe that the smallest possible rectangle has an area of 2 square units. This is a good opportunity to clear this up and show that a square is a rectangle becuase it has four sides opposite sides are parallel and the corners are "square or 90° angles.)	Geoboards Rubberbands of various colors and sizes
Have the children make rectangles of certain areas which you specify. Everyone make a rectangle which has six square units. Have everyone show their rectangles. Some will be 2 x 3, others will be 3 x 2, 1 x 6, or 6 x 1. Have them count and see that each rectangle contains the same number of square units - is the same area or size - even though they all look different. Continue the activity with other numbers. Be sure to accept squares as rectangles. What do they notice different about seven and twelve unit rectangles? (Do not push this but do see if they notice that some areas such as twelve or 16 can be represented by a lot more variation than other numbers such as 5 or 7.)	Geoboards Rubberbands

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (c) Measure water into cups, pints, quarts, and liters.	
Allow children to have free time water activities in which they pour the water back and fourth from one container to another. Before structuring any measuring activities make sure they have all had an opportunity to play with the materials and get an intuitive feel for the various sized containers (That quarts and liters are larger than pints and cups, for example, but not the exact relationship.)	Plastic containers (Size clearly marked) cups pints quarts liters Watertalbe
Ask the children to find out how many cups there are in a quart. Do not direct them in how to find out - let them experiment. At different times ask them such questions as the number of cups in a pint, the number of pints in a quart, how much is a half of a pint, which is larger a quart or a liter, etc.	Plastic containers Watertable
All of the above activities can be done with sand or birdseed instead of or in addition to water.	Sand table Plastic containers

Bilingual First Grade Math Competencies for the Concrete-Operational Child

IV. (d) Use balance scale to find approximate weights of common objects.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
From an assortment of objects (junk box), have the pupils pick objects they believe have about the same mass (from hefting the objects) and then place the objects on the pan balance to see how accurate their estimations were. (This type activity should be used before any actual measuring by standard units is done.)	Pan balance Assorted objects washers paper clips coins beans blicks chains etc.
Have the children take one item from the junk box (preferably something light such as chalk, button, crayon, etc.) and find how many centimeter cubes it takes to balance the object in the pan balance. Tell the children that each of the contemeter cubes weighs one gram. Ask them the approximate weight - in grams - of the object they have balanced. Continue with various sized pieces.	Pan balance Assorted objects Centimeter cubes
After the children have had experience using the one-gram centimeter cubes, add the 10-gram mass pieces so that they can weigh larger objects with less counting. This is good practice in counting by tens (be sure they realize they can add the centimeter cubes if objects do not balance with the 10-gram pieces.	Pan balance Assorted objects Centimeter cubes 10-gram mass pieces
OPEN MATH RESOURCE BOOK: pp. 63, 106-108, 359-360 PRIMARY MATH LAB: pp. 3-37	

Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (e) Read time to the nearest hour.	
Use play clocks. Set the clock at different hours. Talk about what we might be doing when the clock shows that time. (eg. 12:00 - we might be asleep if it is night or eating lunch if it is day.) Have the children think of an activity and then set the clock to the hours that they might be doing the activity. WORKJOBS I pp. 228-229	play clocks

Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (f) Identify pennies, nickels, dimes, quarters: Give their equivalent amounts in pennies.	
Play store: Set up a center with objects priced with amounts from 1¢ to 25¢. Have an older student or an adult work as cashier until the students learn to identify the coins and their value. Give each student a set amount of play money with which to shop. As the pupils develop skill in identifying coins and counting their own change, allow them to also work as cashier in the play store. (If real objects are used in the store then coins might be earned in the classroom in some manner - or, extra coins might be earned over an above a set amount received by each student.)	Play money (DLM) Assorted objects Price tags Cash Register
Coin dominoes: These can be purchased commercially or made by the teacher using blank playing cards (or tagboard strips) and rubber stamps of coin faces (see DLM). The object of the game is to match dominoes with the same amount of money shown (eg. a dominoe end with one nickel and five pennies could match with a dominoe showing one dime or with one showing ten pennies.)	Coin dominoes OR Blank cards Coin Rubber Stamps
WORKJOBS I pp. 224-227	

Set Inclusion

(Addition and Subtraction Readiness) •*Purpose:* This activity evaluates a student's understanding of the concept of a whole in relation to its parts.

•*Materials:* Two kinds of flowers (e.g., daisies and carnations)—seven of one (daisies) and three of the other (carnations).

•*Procedure:* Ask the child whether the daisies are flowers and whether the carnations are flowers. Then ask, "If you were to make bouquets, which would be larger, the bouquet of daisies or the bouquet of flowers?" Children under seven will generally indicate that the bouquet of daisies would be larger. They cannot simultaneously consider the *whole* set of flowers and at the same time distinguish between the *parts* (daisies and carnations). The child who is unable to deal with set inclusion is unable to *reverse* the process by breaking up a whole and realizing that a recombination of the parts is once again equivalant to the whole. This is analogous to a child's knowing that 3 + 5 = 8 but not recognizing that 8 = 3 + 5. Around the age of seven, children are able to consider both the whole and its parts simultaneously and can therefore solve the set-inclusion problem.

Before children have acquired a thorough understanding of set inclusion, it is not advisable to introduce the operation of addition and subtraction at the formal (symbolic) level. When addition is introduced before a child has gained competence with set inclusion, any learning may be rote in nature and true understanding of the concept may be delayed.

Souviney, R. *et al.* *Mathmatters.* Goodyear, 1978.

ACTIVITIES FOR LEARNING THE COMPETENCY — MATERIALS

V. (a) Use real objects to add and subtract numbers with sums to 18.

NOTE TO TEACHER: One reason children have so much trouble with subtraction in later grades is that they learn only the "take-away" model for subtraction in the earlier grades. In real life (and in story problems) subtraction is almost always the "comparison" model and is more often the "missing addend" model than it is the "take away" model. PLEASE be sure that you use examples of more than one model when teaching the concept. DO NOT let the children confuse TAKE AWAY WITH SUBTRACTION.

Take away model activities:

Use real objects to divide sets into subsets; remove one subset; verify the number remaining. The children may count at the beginning to find the number remaining. Follow by giving several different problems in a row (using different objects) which use the same numbers (eg. 7 - 3 = 4). After two or three of these problems, ask the children to predict before counting the amount that will be left the next time. "Do you suppose that when we take 3 from 7 there are always four left? Why?" This leads to the idea that if the answer if always going to be the same for certain combinations we can just remember the answer instead of having to count out objects each time.

Numerous sets of objects

(eg. blocks
buttons
paper clips
pencils
crayons
dishes
dolls
toys
books, etc.)

Comparison model:

The Cuisenaire Rods are an ideal material for introducing this model of

V. (a) cont. Use real objects to add and subtract numbers with sums to 18.

subtraction. Children find THE DIFFERENCE between the yellow rod and the brown rod by laying the yellow rod on top of the brown rod and finding that the light green rod will fill up the amount of space that is left.- or the difference. They can also fill up the amount of space with three white rods. Given the rule white = 1, then, they can discuss the idea that the difference between 5 (yellow) and 8 (brown) is 3 (light green).

Centimeter cubes, joined together, can be used as an example of this model. Make two rods of different lengths from the cubes. Have the child compare the rods and determine which is larger. "How many more cubes does this rod have? What is the difference between the two rods?" This can be a bridging activity from the take-away model, since the child can take the extra cubes from the longer rod before counting them.

Missing Addend Model:

A pan-balance and centimeter cubes (or other items of a uniform mass) can be used to model the missing addend. Place five cubes on one side of the balance and two cubes on the other. Ask "How many cubes do I need to add to the two cubes to make the pans balance?" Let the children add them one at a time if they cannot predict the correct amount. "Two plus how many was five?"

A number balance is also an excellent model for this, however, most first graders seem to think of it as a magic trick. It is better to use it at later grades.

Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
V. (a) cont. Use real objects to add and subtract numbers with sums to 18.	
Place a number of dissimilar objects on a table (dissimilar so that children will not develop the idea that the members of a set must have some characteristic in common other than that they are members of the same set). Ask children to name the members of the set (eg. a block, a pencil, a key) as each is pointed to. The set will be well defined if the children can answer such questions as these: "Is the block a member of the set?" (yes) "Why?" (It's there.) "Is the elephant a member of the set?" (no) "Why?" (It's not there.)	several objects (one of each) (eg. pencil block key eraser, etc.)
Use the flannel board to develop the idea of joining sets to find the number of a new set and begin work on the commutative property of addition. Ask the children, "What things do you see in each set?" How many members are in the first set? How many in the other set?" Move the set with an apple and duck (for example) to join the other set (eg. bunny, star, pear) and ask, "If you join a set of two with a set of three, how many will be in the new set?" Separate the sets and then join the set of three with the set of two and ask the same question. Present several different joinings, using numbers other than zero. The children may need to count to determine the number of the new set. After this routine has been established, use different objects but the same number of objects three or four times in a row, (eg. 4 and 3). After these numbers have been used several times, take out a new set of 3 and a new set of 4 and ask the children to predict how many members the new set will have after they are joined. "Do you suppose 3	Flannel board Felt pieces of many sizes and shapes

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

V. (a) cont. Use real objects to add and subtract numbers with sums to 18.

---

objects and 4 objects will always join to make 7 objects?" Let them try it many more times if they are not sure. This is an important step because it establishes the idea that if the sums are always the same we can just remember the answer instead of counting out objects each time we need to know the total of 4 and 3. (The above activities may be done with a smaller group using real objects on trays or tables instead of the flannel board.)

WORKJOBS 150-151, 174-175, 194-211

WORKJOBS II - all activities

ADDITION AND SUBTRACTION (MAKING QUANTITIES EQUAL)

Purpose of Activity

The purpose here is to study children's ability to make use of addition and subtraction to solve problems involving unequal quantities.

Materials Needed

Two sets of counters, one of 8 and one of 14.

Procedure

Show the child the two sets and ask her to make them equal.

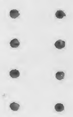
Levels of Performance

Stage 1. At this stage, from 4½ to 6 years of age, the child does not understand that addition and subtraction compensate each other as a reversible system. As she adds counters to the smaller set, she is not aware that she has reduced the larger. She may not even use the logic of number and simply make a global comparison—"They are the same because they 'look' the same size" (occupy about the same amount of space).

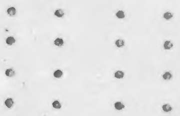
Stage 2. The child at this stage has an intuitive grasp that in adding to one set she is subtracting from the other, but equalizing the sets is still a trial-and-error process. As she moves the objects from one set to the other, she judges by physical appearance. She may then count to make sure.

Stage 3. Children aged 5½ to 7 have an immediate grasp of the necessary addition and subtraction. They realize that 8 and 14 can be equalized by adding 3 of the 14 to the 8, that the 8 becomes 11, and that 14 − 3 also becomes 11. They solve the problem at the numerical level without being tied to the physical arrangement of the counters as are the Stage 2 children.

For example, both a Stage 2 and a Stage 3 child begin by arranging the set of 8 as follows:



They then begin arranging the set of 14 in the same pattern beside the set of 8.



The 6 left over are divided by placing 2 more in each set and then 1 more in each set.

The difference between the Stage 2 and the Stage 3 child is that the former is tied to the physical layout, making judgments *after* the division is made and relying on how the figures look after the division. If the shape of one of the sets is changed, she is not sure of the equality. Hers is not a numerical division, but a physical one.

The Stage 3 child, on the other hand, knows from the beginning that if one set is larger than the other, she must therefore divide the difference (6). This she does at the numerical level and then demonstrates by arranging the counters.[9]

Teaching Implications

This activity identifies children who have an operational understanding of addition and subtraction as contrasted to those attempting to solve such problems without the necessary mental operations.

As summed up by Piaget, "numerical addition and subtraction become operations only when they can be composed in the reversible construction which is the additive 'group' of integers, apart from which there can be nothing but unstable intuition."[10]

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[9] Jean Piaget, *The Child's Conception of Number* (New York: W. W. Norton & Company, Inc., 1965), p. 194.

[10] Ibid., p. 195.

Copeland, R.W. Math Activities for Children. Merrill, 1979.

THE COMMUTATIVE PROPERTY OF ADDITION

Purpose of Activity

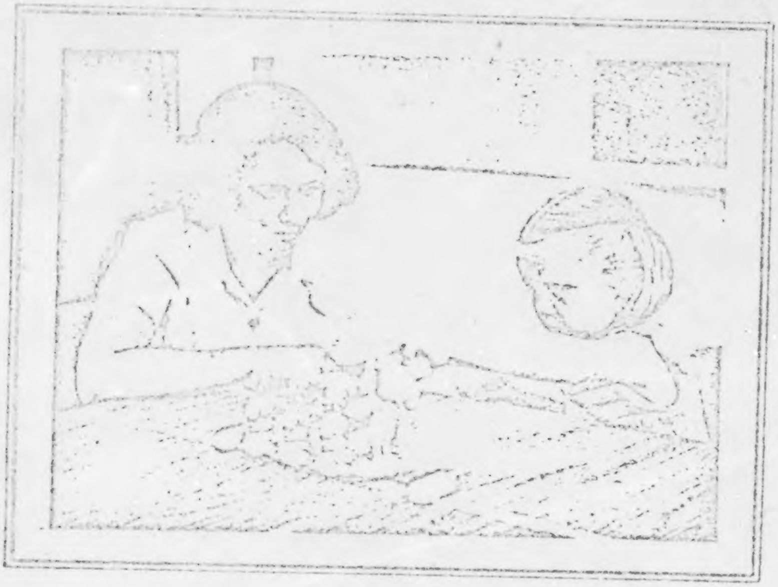
What is children's understanding of the commutative property of addition—that the order of adding numbers does not change the sum? For example, when do children realize that 2 + 7 = 7 + 2?

Materials Needed

Two sets of objects, such as 2 yellow toy cars and 7 blue cars.

Procedure

Ask the child to line the cars up in a row with all the cars of one color followed by all the cars of the other color. Put a parking marker at both ends of the row. Then remove the cars from the row. If the

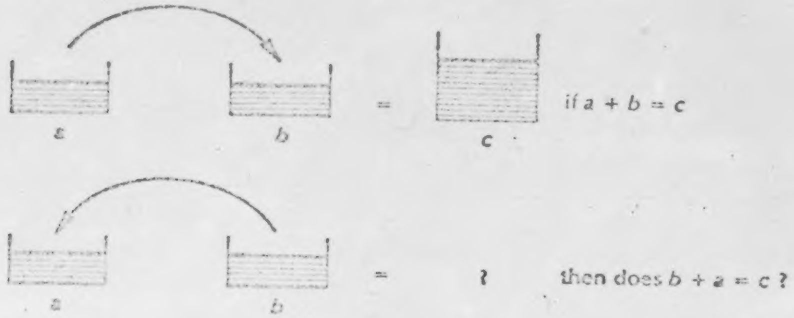


Checking to see if a row of two "Mickey Mouse" dolls followed by three dwarfs is the same length if the order is reversed.

child began his row with yellow cars, call that to her attention and ask her to do it again beginning with the blue cars. After she has placed two or three cars, stop her and ask, "Do you think this row will reach to the same parking marker, or not as far, or farther? Why?"

A variety of other materials could be used for the same purpose—dolls of two types, for example.

Also, to investigate the idea further, use continuous sets, such as liquid to which food coloring has been added.



Levels of Performance

Children will be successful with these tasks on the average at 7 to 8 years of age. The tasks involve the concept of reversibility which develops around 7 years of age.

Teaching Implications

Present practice of teaching addition in the first grade ignores the necessity of reversible thought as a basis for understanding the operation. It is one thing to teach that 3 + 2 is equal to 5 and that 2 + 3 is equal to 5 and have children memorize that the response is 5 in both cases; but the activity just presented demonstrates that teaching the concept is not the same as teaching a response.

When children *do* have reversible thought, the commutative property simplifies teaching the 100 basic addition facts, because for every addition fact such as 7 + 2, they also know 2 + 7.

Copeland, R. W. Math Activities for Children. Merrill, 1979.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
V. (b) Recall addition facts through sums of 10.	
Keeping a number in your head: Make a special die with felt pen and sponge cube, using the numerals 0 through 5 on each face. Each child rolls the die and reads the number. He remembers that number "in his head" and rolls the die again. He must add the second number to the first number. He may count out markers for the second number but not for the first. (Example: child rolls a 3 the first roll and a 4 on the second. He remembers the 3 and then counts out 4 markers. He counts the markers by first saying 3, then counting 4,5,6,7. He gets one point for getting the right answer.) Play until one player earns five points. (Although more than 2 players can play, pairs are better because each player gets his turn more often.) See pages 3 to 17 of the book Addition and Subtraction Games and Ideas by Gary Bitter and others. Each page has at least one game or activity that meets this objective.	Sponge cube * Felt marker Markers *For each pair of students.

Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
V. (b) Recall subtraction facts through sums of 10.	
Use various board games with two different color dice (one labeled with numerals 0 - 5 and one with numerals 5 - 10). Children throw the dice, find the difference, and move that number of spaces on the board. (Also can be used with a deck of flash cards containing all of the facts to be learned.)	Blank game boards dice cards
Make a deck of cards with pairs of cards; one containing a subtraction fact and the second the difference. Have the children use the cards to play war. Add one odd card and they can play "Old Maid".	Blank cards of different colors Felt mark pens
Make a Slap game with differences from 0 through 9 and problems with minuends of 10 or less.	
WORKJOBS: pp. 150-151, 194-195, 210-211.	
WORKJOBS II: all activities	

Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
VI. (a) Make picture graph with information student helped to gather.	
Suggested graphing activities for picture graphs:	
Children's birthdays - Have each child glue a picture of a birthday cake next to the month in which their birthday occurs.	cut out pictures of birthday cakes (or gummed stickers) Large chart with months of year printed down left side. (glue)
Children's pets - Have each child glue a picture of a dog, cat, fish, bird, etc., next to the corresponding label to show which kind of pet they own.	cut out pictures of pets (or gummed sticker Large chart with names of pets down left side.
Alternative: Instead of using tagboard and glue for the above activities, have the children form graphs on a flannel board with the captions and pictures made from felt.	
Open Math Resource Book: pp. 148-150 WORKJOBS: pp. 222-223	

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
VI. (b) Make bar graph with information student helped to gather.	
INTRODUCTORY ACTIVITIES - USING BLOCKS	
Have every child who walks to school stand up. Give each child a red block (interlocking blocks such as unifix cubes are especially good for this). Continue activity with different colored blocks for persons who come to school in different forms of transportation.	Colored blocks or Unifix cubes Sentence strips felt pen
Write or draw a picture of each category on the sentence strip and place it on the table Have the children stack their blocks on the word or picture for their color. The result is a physical representation of a bar graph.	
After one or two activities with physical graphs, have the children "draw the graph" on graph paper so they can use the blocks for some other purpose and still keep their recorded data.	
M & M's	
Give each child one small package of M & Ms and a sheet of c^2 graph paper. Introduce the word "mode" as a word in statistics that represents the largest set in a collection of sets. Your object is to find the "mode" for M & M colors. Have each child place the M & Ms from their package on their own sheet of graph paper so that all of one color form a line - one piece per square. The longest line was the "mode" for that package. Do they all have the same mode?	Small package of M & Ms per pupil Graph paper (centimeter squares) Crayons
Now take out crayons. The children may eat the M & Ms on their papers one at a time	

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
VI. (b)—cont. Make bar graph with information student helped to gather	
but as they remove the candy they must color in the square with the color of the candy to keep track of what they have eaten.	
PLANTS	
Have children plant seeds and once sprouted, keep a daily record of the plant's growth. Children can tear or cut a piece of paper or string the heighth of their plant and glue it on a chart with a base line and the date.	seeds planting medium individual charts string or paper strips (adding machine tape is very good)
OPEN MATH RESOURCE BOOK: pp. 148-150, 227-228.	
WORKJOBS: pp. 222-223	
PRIMARY MATH LAB: pp. 57-60	
Suggestions for further graphing activities: Favorite T.V. shows Color or make of family auto Color of socks worn to class on a particular day. Favorite Soft drink Favorite school lunch entre Favorite playground activity.	

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS

VII. (a) Identify basic shapes of circle, triangles, rectangles, and squares.

See Pre-operational activities V. (b)

Introduce the square as a special type of rectangle.

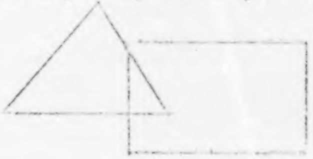
Continue finding shapes in the environment.

SIMON SAYS:

Make 2 large overlapping shapes on the floor with masking tape or yarn.

- masking tape or yarn

example:



Children take turns playing Simon Says (with a teacher, aide, or older student as the leader).

example: Simon Says - Step on the triangle.
- Step inside the rectangle.
- Step on both the triangle and the rectangle.
- Step inside both the triangle and the rectangle.
- etc.

Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
VII. (b) Construct basic shapes on geoboards.	
Have children begin by making any shape and then describing it. Have them make designs and then identify particular shapes in the design. Then have children make prescribed shapes (no size requirement) and compare the various interpretations. Have the class as a whole copy shape from one board. Play a challenge game where one child makes a shape and another child must copy it exactly. PRIMARY MATH LAB: pp. 50-55	geoboard for each pupil rubberbands of different colors

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IX. (a) Put objects into a sequential pattern.	
See pre-operational activities number VI. (a)	

ACTIVITIES FOR LEARNING THE COMPETENCY — MATERIALS

IX. (b) Describe the pattern created with objects (include quantitative terms).

Use activities previously described for creating patterns (pre-operational number VI. (a)). Have the child verbally describe the pattern and tell which material goes next. (This can be done first with teacher, aide, or older student in a small group. Then it should be done by pairs of pupils where one creates a pattern and the other describes it and repeats it.)

FOLLOW UP ACTIVITY AT THE ABSTRACT LEVEL:

After the students have had many experiences with the concrete objects in patterning and describing the patterns, they may begin to use pictorial patterns with basic shapes which they can easily draw. Also, a pattern of colors can be completed and described where the shapes are all the same.

WORKJOBS: 26-27, 36-37, 184-185, 190-191

OPEN MATH RESOURCE BOOK: pp. 51-60

MATERIALS

beads & string
Cuisenaire Rods
blocks
pasta (dry)
shells
construction paper (multicolored)
attribute pieces

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

IX. (c) Complete number patterns of basic counting by ones, twos, fives, and tens.

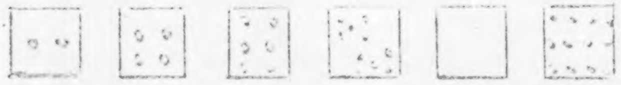
ORALLY

Begin a counting sequence (eg. 2,4,6,8,...) and then stop, signalling the child to continue the sequence.

WRITTEN

Use a dominoe type dot system to illustrate a sequence with a few blank squares inserted (one or two only when beginning - if two, not adjacent).

Leave blank spots in numerical patterns and places at the end so that pupils can discover the pattern and continue it.



5,10,15,20,___, 30,35,___,45,50,___.___,___,

Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
X. (a) Identify one-half, one-third, and one-fourth of such things as a cup of water and of the basic geometric shapes.	
Have children use measuring cups to experiment with fractions. Using the $\frac{1}{2}$ cup and the cup, pour various $\frac{1}{2}$ cups (water, rice, sand, etc.) into the cup. How does the cup look when you have put in $\frac{1}{2}$ cup? How does it look when you put in 2/2 cups? Do two half cups always equal the full one cup? Continue with 1/3 and 1/4 cups.	Measuring cups (dry measure type) sand water rice, etc.
PRIMARY MATH LAB: pp. 63-71 (cooking)	
Use felt fraction kit and flannel boards to allow children to explore congruent fractional parts of a whole.	flannel board felt fractions - circle felt fractions - square

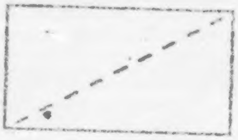
(Fraction Readiness)

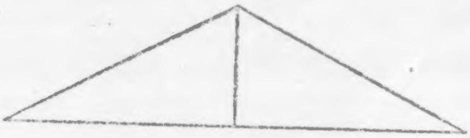
Conservation of Area

•*Purpose:* This activity evaluates a student's understanding of the invariance of a concrete object's area when the object is divided into pieces.

•*Materials:* Two equal cardboard rectangles, one cut diagonally to form two equal triangles.

•*Procedure:* Place the two triangles on the rectangle so that the triangles fit the surface of the rectangle. Once the child has agreed that these areas are equal (i.e. that the same amount of paint would be required to cover the triangles as to cover the rectangle) remove the triangles and arrange them as below.





Ask the child whether the two shapes are still the same size (i.e., whether they would require the same amount of paint to cover them). Most children under the age of seven will say that the second figure is larger. They are guided by their perception: the second figure has more pieces or simply appears larger. Between the ages of seven and eight, most children develop the notion that cutting and moving the pieces about does not change the total area. They are able to mentally reverse the procedure in order to justify their conclusions. Since the definition of fraction is often based on the area concept, formal work with fractions should be delayed until after understanding of area conservation is developed. Introducing informal investigations with fractional parts and fractional parts of sets of objects is appropriate only if conservation-of-number concepts have been mastered.

Souviney, R. *et al*. *Mathmatters.* Goodyear, 1978.

FRACTIONS

Purpose of Activity

The purpose is to explore children's understanding of the quantitative relations involved in dividing a whole into equal parts.

Materials Needed

1. Paper cut into various shapes, such as circles, squares, and rectangles.
2. Six dolls.

Procedure

Show the child a circular piece of paper and two dolls and ask, "If this were a cake, could you cut it so that each doll would get the same amount? Can you show me with this pencil how the cake should be cut."

Vary the procedure with other shapes, such as a square and a rectangle.

Also use the same procedure with three dolls, then six dolls.

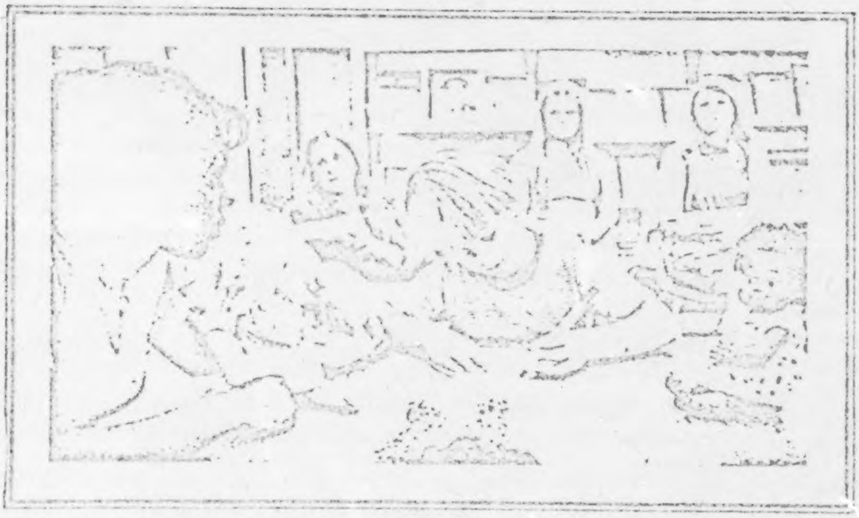
Levels of Performance

Stage 1. In children under 4 years of age, there is no plan and the divisions are not equal. The child is unable to relate the part to the whole once the cake is cut. The parts are now less or more than the original cake.

Stage 2. At Stage 2A, 4 to 6 years, children can divide small cakes into two equal parts, but not into three equal parts.

At Stages 2B and 3A, 6 to 7 years of age, children can divide into three equal parts, but it is a trial-and-error process; for example, drawing a line and then erasing it to redraw the line in a different position. These children do understand conservation of the whole—that once the cake has been cut it is still the same amount of cake.

Stage 3. At Stage 3B, around 10 years of age, children can with



Dividing a "cake" for three dolls.

some assurance tackle the problem of dividing the cake into six equal parts. They may, for example, first divide it into three equal parts and then divide each of the three into two equal parts.

Teaching Implications

Children are often introduced to fractions by being "shown" something divided and "told" the name of each part. Then they are given pictures divided into parts and asked to name the parts.

Children should do the dividing as well as naming the resulting fractional part. As this activity demonstrates, it is not as easy as one might think.

Copeland, R. W. Math Activities for Children. Merrill, 1979.

Bilingual First Grade Math Competencies for the Concrete-Operational Child

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
X. (b) Handle and count fractional models (eg. egg cartons or fraction bars) to amounts larger than one whole (eg. five thirds)	
Have each child paint their own set of egg carton fractions which you have cut. Establish a color pattern and display it. All fractions of the same size must be the same color (Note: use only halfs, fourths, thirds, and wholes. Make both models of $\frac{1}{2}$. If using the 30 egg carton with 24 as a whole, make both models of $\frac{1}{4}$ also.)	egg cartons (12 or 30 egg size) knife or cutting board 4 colors - tempra paint
Hand a child a $\frac{1}{2}$ (egg carton or fraction bar) and ask him/her what she/he has. Hand him/her a second $\frac{1}{2}$ and repeat the question (answer 2/2). Hand him/her a third $\frac{1}{2}$ and repeat the question (answer 3/2). Continue. Bring out models for one whole and find how many wholes are the same size as the pupil's halfs. Establish the concept of different names or labels for the same amount or number. (Also introduces pupil to the idea that we use fractions to talk about amounts greater than one whole.)	egg carton fractions or

Bilingual Second Grade Math Competencies for the Concrete-Operational Child

I. Classification (Logic) /Comparing/Ordering.

"The student will have experiences classifying, comparing, ordering; the use of number and reversability allows the student to extend these activities through numerical descriptions."

(a) Classify objects by size, color, and the four basic geometric shapes (circles, squares, rectangles, triangles) and classify sets by the number of members.

(b) Compare sets of objects and verbalize about their size.

II. Numeration

"The student will have experiences with numeration; extensive use of numeration models will be used with many grouping and trading activities for developing the place value concept."

(a) Group bundles of over 100 popscicle sticks into 100's, 10's, and 1's.

(b) Participate in activities using manipulative which are based on size but require trading rather than bundling.

(c) Participate in activities using manipulative with arbitrary classification (not based on size).
(eg. colored chips, soda pop bottle lids, etc.)

(d) Count forward to 120 by 1's, 2's, 5's, and 10's and backwards from 20.

(e) Use zero as a place holder.

III. Notation

"The child will learn the written symbols associated with numerals, operations, and measurements after having concrete experiences with the concept and developing an oral label for the concept.

(a) The child will read and write the numerals from 0 to 9.

(b) The child will read and write the numerals from 10 to 999.

(c) The child will read and write the symbols for "greater than" "less than" and "equal to."

(d) The child will read and write the equations for the basic facts with sums to 18 in addition and subtraction (numerals, operation signs, equal sign).

(e) The child will begin writing number sentences expressing inequalitie with numbers of nine and less (numerals, operation signs, greater than and less than signs).

(f) Pupil will write numerals from 0-999 when shown a manipulative model of the number.

(g) Pupil will write numerals from 0-999 from oral dication.

IV. Measurement

"The student will have experiences with measurement; early experiences with non-standard measurement will be followed by many activities at every level using both the metric and English systems for length, mass, capacity, area, and temperature. The measurement of time will include sequencing, and many activities of short (hours, minutes, seconds) and long (day, week, month, season, year) duration.

(a) Measure objects, using ruler, yardstick, meterstick, and tape measur

(b) Determine area of a rectangle using a geoboard.

(c) Measure water into cups, pints, quarts, and liters.

(d) Child will weigh self to nearest pound and (if scales available) to the nearest kilogram.

(e) Read time to the nearest ½ hour.

(f) MONEY: Identify any combination of coins up to $1.00 and record using both $ and decimal ¢.

V. BASIC Facts (Addition and Subtraction of Whole Numbers)

"The student will have experiences with basic facts; once a child is able to demonstrate an understanding of an operation through the use of concrete materials, drill activities for memorizing the basic facts will be implemented."

(a) Use real objects to add and subtract numbers with sums to 18.

(b) Recall addition facts through sums of 18.

(c) Recall subtraction facts through sums of 18.

VI. Algorithms For Operations

"The student will have experiences with algorithms for operations; introduction for multidigit numerals via concrete materials and to be taught as a way of recording the concrete experience."

(a) Use real objects to add and subtract multi-digit numbers.

(b) Record multi-digit addition and subtraction problems from manipulative model.

(c) Solve multi-digit addition and subtraction problems using paper and pencil (using either an intermediate or standard algorithm).

VII. Graphing

"The student will have experiences organizing and presenting data; quantities will be graphically represented in numerical form as well as pictorial."

Students will prepare a bar graph from data they have collected and write at least one paragraph interpretting their results.

VIII. Geometry

"The student will have experiences with shapes and space - extensive work with geoboards in exploring measurement of perimeter, area and angles."

(a) Identify 3 dimensional shapes of sphere, cylinder, cube, retangular prism, triangular prism, and pyramid.

(b) Use pattern blocks and mirrors to create a tessalating design.

(c) Create and outline designs with tan grams for their peers.

IX. Patterns

"The student will have experiences with patterns; patterning will be continued with concrete materials (e.g. attribute blocks) but will now include numerical patterns as well."

(a) Explore number patterns involving both addition and subtractio

(b) Use colored rods to make stair cases of odd and even numbers.

X. Fractions

"The student will have experiences with fractions; early concrete experiences will develop the language of fractions (including physi adding fractional parts with the same name and naming the new quant Numeration will be introduced once the child can divide."

(a) Identify one-half, one-third, one-fourth, one-sixth, and one-e of such things as a cup of water and of the basic geometric sh

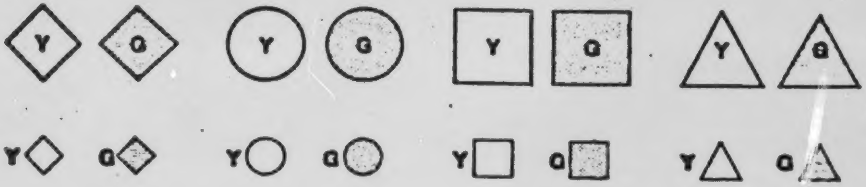
(b) Handle and count fractional models (eg. egg cartons or fractio bars) to amounts larger than one whole (eg. five thirds).

Logical Classification

(Number Readiness)

•*Purpose:* This activity evaluates a student's ability to sort (classify) objects consistently based on observable likenesses and differences.

•*Materials:* A set of shapes similar to those below. Each shape should be provided in two colors.



•***Procedure:*** Have the student dump the blocks onto the table and ask him to sort them into two groups so that each group is alike in some way. Children who do not classify logically will have difficulty with this problem. They may change their grouping criteria in the middle of the problem or may simply make pretty patterns with the shapes. In most cases, children between the ages of two and six fall into this category. Around age six, children generally become more adept at solving this problem but nevertheless require some help with organization in order to ensure that all the blocks will fit into one group or the other. Between the ages of six and eight, most children are able to solve this problem independently and without hesitation, providing complete descriptions of their grouping criteria.

Until a child can classify logically, it is not advisable to introduce number concepts at the abstract level. Number concepts should remain implicit in the concrete materials and not formally defined by means of symbols.

Souviney, R. et al. Mathmatters. Goodyear, 1978.

. (a) Classify objects by size, color, and the four basic geometric shapes (circles, squares, rectangles, triangles) and classify sets by the number of members.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
Workjobs 144-145	
Give children a set of attribute blocks and tell them to sort the blocks into two groups (give them two boxes) so that every thing in each box is related.After they have finished ask "How are these alike?" "Can you sort them into two boxes again but in a different way?" (Continue this until they have sorted by all double attributes such as large and small, thick and thin, round and not round.) Next give them the number of boxes that are equal to the number of colors and shapes in the set and ask them to continue grouping. Always ask how the groups are chosen, "How are these alike?" (If attribute blocks are not available assemble a collection of cars, pencils, and blocks - some red, some blue, some yellow for instance, and some larger than others. Also, attribute pieces may be made from colored construction paper and laminated, however, they will lack the attribute of thick and thin.)	Attribute pieces
Bring out a variety of objects. Begin to put the objects into two sets, perhaps on a piece of tagboard marked into two sections. Into one set put things with a common property or identity - animals, perhaps - and into the other set objects that are not animals. As soon as they think they know it, have the children tell the name of either set. They will easily name the set of animals; the difficulty comes in finding a name for the other set. They may suggest names like "things" or "toys" or "objects", but if any of these were the name of the set then all the animals would be members of that set, too, and they are not. If no satisfactory name can be suggested, put that display aside, still in view, and begin another partitioning, such as "yellow" and "not yellow". Again seek names for the two sets. If several such examples do not elicit the desired response from the children, return to the set of animals and say, "Listen while I think how I'll decide where this ones goes. Is it an animal? Yes; so I'll put it here with the set of animals. Is this an animal? No; so I'll put it with that set. This is the set of animals. What could be a name for that set?"	Numerous small objects
Display sets of objects whose only common attribute is the number of members (eg. 5 gold stars, 5 red balls, 5 blue rectangular regions, 5 pink erasers, 5 yellow pencils, etc.) Ask the children to tell you what all of the sets have in common. (If all other attributes have been controlled, you would have no trouble eliciting the attribute of number form a group of children, however one child alone may not be able to recognize number as an attribute.)	Sets of small objects (eg. gold stars, red balls, blue rectangular regions, yellow pencils, silver jacks, etc.)

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
I. (b) Compare sets of objects and verbalize about their size.	
In a small group setting, give the children a box of pencils (or similar objects) and ask whether there are more red pencils or yellow pencils. How many more? Are there enough pencils for each child in the group to have one? Could each boy get a red pencil and each girl get a yellow pencil? Such questions focus on the relative sizes of sets of objects and encourages children to verbalize about them. This activity can assist the teacher to identify the students who can merely repeat the names of numbers that they have memorized in rote fashion with no true understanding of their relative size and order.	Box of pencils (or other objects)
Have the children sort sets of objects so that in one location are sets with, for example, eight members, in another location are sets with fewer than eight members, and in a third location are sets with more than eight members. Ask the children to explain to other children what the rules for sorting the sets are in their own words.	numerous sets of obje (eg. blocks buttons dolls dishes books toys)
Show the children a set and have them find another set that has one (two or three) member(s) more. "How do you know it has one member more? Show me."	numerous sets of ojbe
Display pictures and have the children put together those pictures that show sets with the same number. "How did you know these went together? Show me. Show (name of another child) how to do this activity."	pictures of sets (some of the same num of members and others different)

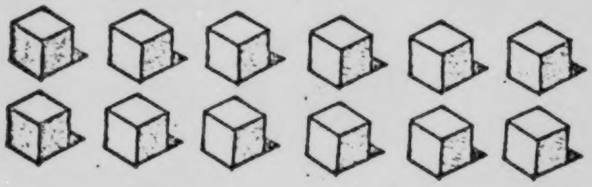
Conservation of Number

Purpose: This activity evaluates a student's understanding of the invariance (conservation) of the number of objects in a set when the elements are moved about.

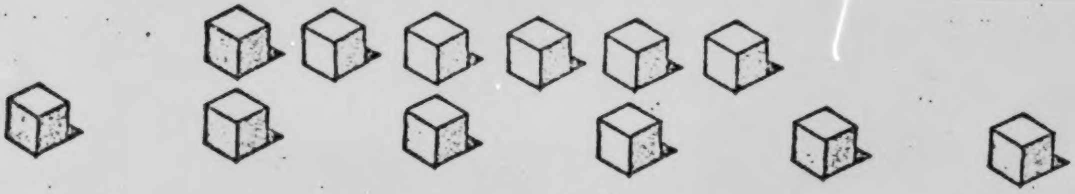
(Number Readiness)

Materials: Twelve identical wooden cubes.

Procedure: Make a row of six cubes in front of you. Give the rest of the cubes to the child and ask him if he has the same number of cubes as you have. Have him prove it by putting the blocks in one-to-one correspondence, as below.



Now spread his blocks out and ask him whether you both still have the same amount.



Young children (four to five years of age) have difficulty determining whether the numbers of blocks are equal and, generally cannot place the two sets in one-to-one correspondence in the first place. Even if they can initially determine equivilence by putting the two sets into one-to-one correspondence, when the rows are transformed their decisions are ruled by perceptual considerations; therefore, they believe that the spread-out row contains more cubes. Over the next two years, they will become capable of putting the blocks in one-to-one correspondence, but the decisions will still be ruled by perceptual appearances. By the age of seven, children are usually able to solve this problem correctly. We then say that the child conserves number.

As with classification and seriation, it is not recommended that children be introduced to number concepts at the abstract level until they conserve number. Once children are dealing effectively with classification, seriation, and conservation of number, they are ready to be introduced to the abstract notion of number and the associated symbols.

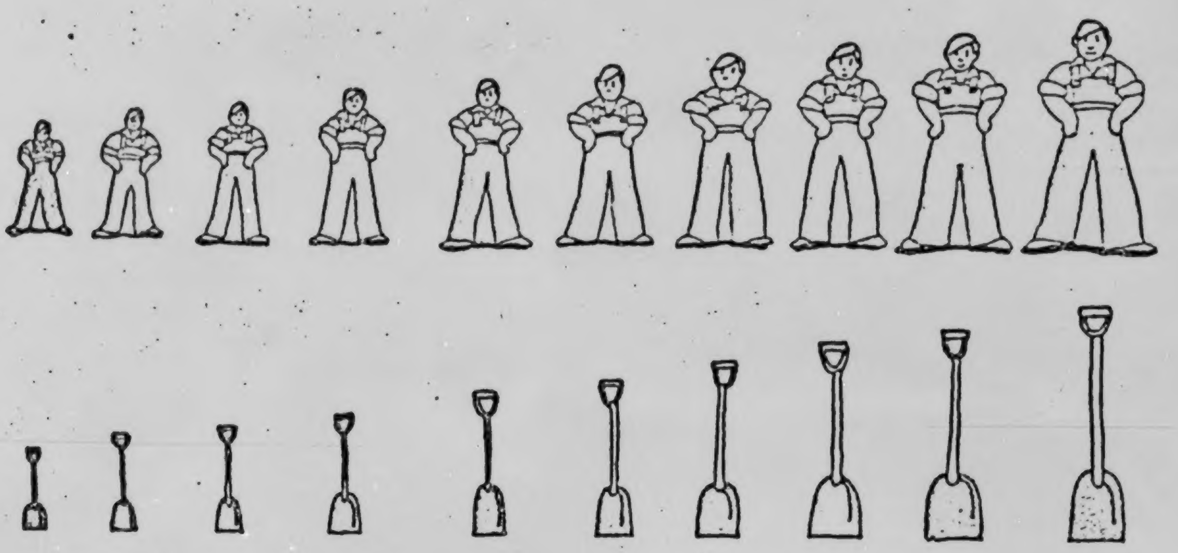
Souviney, R. *et al*. *Mathmatters*. Goodyear, 1978.

Seriation

(Number Readiness)

•*Purpose:* This activity evaluates a student's ability to place objects in serial order based on some attribute of size.

•*Materials:* Ten paper cutouts of workers gradually increasing in size, with the largest twice as big as the smallest. In addition, you will need ten paper cutouts of shovels gradually increasing in size, with the largest three times as big as the smallest.



•***Procedure:*** Ask the child to arrange the workers in order from small to large. Then have him position the shovels so that each worker gets the right-size shovel. Mix the workers up and ask the child to rearrange the shovels accordingly. Next give him the shovels, point to one of the workers, and ask the child to produce the appropriate shovel.

Young children (two to five years old) are unable to seriate either the workers or the shovels. Over the next two years, most children develop the ability to solve the three seriation problems, primarily by trial-and-error methods. Generally abilities necessary to solve the three problems are acquired in the order in which the problems have been listed above. After age six, most children readily solve all three seriation problems without using trial and error.

Until a child can deal effectively with concrete seriation problems, number concepts should not be introduced at the abstract (symbolic) level. Seriation is very closely tied to the concepts of cardinal and ordinal numbers and should therefore be prerequisite to the formal introduction of these concepts.

Souviney, R. et al. Mathmatters. Goodyear, 1978.

ACTIVITIES FOR LEARNING THE COMPETENCY | MATERIALS

(a) Group bundles of over 100 counting sticks into 100's, 10's and 1's.

Banker's Bundling Game (from A.S.U. lab manual) 4-6 players

1) Players each role a dice. Player with highest # is designated as banker and takes charge of the counting sticks.

2) Player to the left of the banker begins. Player throws a dice, asks banker for that # of ONES, and bundles when possible. (Trading banker 10 ones for a bundle of 10 speeds up game but provides less practice in bundling sticks.)

3) Play continues in this manner, each player except the banker in turn throwing a dice, receiving ONES, and bundling where possible.

4) The winner is the first person who makes a HUNDREDS bundle. Winner becomes banker if played again.

5) VARIATION: Each player begins with a HUNDRED bundles and discards the number of sticks thrown on dice, unbundling when necessary. Winner is first player rid of all sticks.

counting sticks
(in bundles of
1, 10 and 100)

dice
rubber bands

I. (b) Participate in activities using manipulatives which are based on size but require trading rather than bundling.

Materials which meet these criteria include Base-Ten Blocks, number blocks; Diene's Multibase blocks; Cuisenaire Rods, Squares, and cubes; Bean Sticks; and tile strips. The latter two are "home made" manipulatives as follows.

Bean sticks are student constructed materials created by gluing exactly 10 dried pinto beans on a tongue depressor. Ten sticks are glued together into a raft of 100 beans. Single beans are used for ones.

Tile Strips are student constructed materials much like bean sticks. Small craft-type ceramic tiles (or large 1" tiles if necessary) are glued onto strips, 10x10 arrays are glued onto squares using rows of alternating colors for easy counting by tens. (Large tile outlets generally have cases of off-color tile or tiles which have come loose from their cloth backing which they will sell cheaply.)

ACTIVITIES:

Bankers game (See II(a)) can be played with any of these materials. The game can be speeded up (and/or played to 1000 using the commercial materials) by having dice of two colors - one for the number of ones and one for the number of tens. (Variation - let children throw dice and choose which will represent ones and which will represent tens.)

Base 10 blocks
Number blocks
Multibase blocks
Bean Sticks or
Tile Strips

See Also:

Mathmatters pp 54-117

Open Math Chapter 2 and 3

Workjobs I pp 146-147

Dice
(or sponge cubes
with numerals written
on each face)

IMPORTANT: Activities in II.(a) are pre=requisite to use of these materials.

(c) Participate in activities using manipulatives with arbitrary classification (not based on size).

MATERIALS

Materials that meet these criteria include commercial Chip Trading Materials, poker chips, a large assortment of lids of particular brands of soda (where coke lids are I, Pepsi lids are 10, etc. Dr. Pepper lids are 100, etc. based on quantity of each brand), construction paper squares of various colors, different types of dried beans (where pinto beans are 1, navy beans 10, lima beans are 100, etc) or any other system that students and teacher can come up with.

Colored chips
pop lids
construction paper
beans
etc.

ACTIVITIES

Activities can be those used in II (a) and (b) but with the new materials.

NOTE: The activities with materials described in II (a) and II (b) are pre-requisite to these materials.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

II. (d) Count forward to 120 by 1's,2's, 5's, and 10's and backwards from 20.

Cuisenaire Rods are good materials to induce counting by multiples. Give the child a stack of red, yellow, or orange rods and the rule that white equals one. Ask the child to count the value of his/her rods using that rule. Unifix cubes in sets of 2, 5, and 10 can also be used in this manner.

Have children stand in pairs (or walk by in pairs)while others count by 2's in unison.

Have children pass out materials to a group, one at a time, telling the number left each time. Eg. If the child has 23 pencils to distribute, she/he gives the first pencil and says "22left", gives the next pencil and says "21 left" etc.

ACTIVITIES FOR LEARNING THE COMPETENCY

MATERIALS

II (e) Use zero as a place holder.

(See activity II (a) for first grade concrete operational. These activities are a continuation and a brief review is probably in order.)

Have children use manipulative materials from II (a), (b), or (c) to record. Begin by asking students to model numbers such as 132 with their material and then to write the numeral on a place value chart:

hundreds	tens	ones
1	3	2

After they have successfully modeled two or three such numbers, ask them to model the number 203 (be sure to pronounce this as two hundred, three - no ands). While recording the number on the chart, ask the children what they should write in the tens space. Solicit the idea of using a zero from the group. If they do not suggest a zero (if they want to leave it blank) do so for the time being. Go to the numbers they have already recorded and ask them to write the numberal without the place value chart. After they have written these numbers ask them to write "two hundred three" in standard form. Ask how we can write "two hundred three" to distinguish it from "twenty-three" (show a model of each). If the idea of using zero is still not volunteered by the group, tell them that this was a problem men had for thousands of years until the zero was invented. Demonstrate the recording of various numbers with a zero in the tens or ones place.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

I. (a) The child will read and write the numerals from 0 to 9.

See II. (c) activities

Activities:

Make 5" x 8" cards for each number from 0 to 9 with the word name for the number on one side, the numeral on the other side, and a pictorial representation under the numeral (eg. seven dots under the numeral 7). Attach the number of clothespins for the number at the top of each card.

5 x 8 cards
felt mark pen
clothespins

Have children remove clothespins from several cards, mix pins, then pull out the correct number of pins to put back on the cards.

Have students copy the numeral and pictorial representation from the card.

Use the black line masters from Mathematics Their Way to practice writing the numerals.

Remember: most numerals face the opposite direction of most letters (alphabet) and children will often reverse the numerals. Make them aware of the problem but accept it as a normal problem of maturation.

[illegible]I. (a) Cont. The child will read and write the numerals from 0 to 9.

Concrete operational pupils still having difficulty writing these numerals at 2nd grade probably need more tactile experiences with the numerals. Have sandpaper numerals for children to trace with their fingers (both with eyes open and again with eyes closed). Make large numerals on construction paper. Have the child glue yarn on the numeral. Give the child interesting ditto papers where pictures of sets of objects are followed with the numeral in dots for the student to trace. Have the child trace the felt numerals on Bell Bags while blindfolded.

ACTIVITIES FOR LEARNING THE COMPETENCY

MATERIALS

[illegible]. (b) The child will read and write the numerals from 10 to 999.

DO NOT HAVE CHILDREN WRITE NUMERALS UNTIL THEY CAN COUNT OUT THE NUMBER IN OBJECTS.

Activities:

Once children can write the digits from 0 through 9, the writing of multi-digit numerals is primarily a matter of spacing. Have children turn their lined papers side ways, make one digit in each space, and leave one or more spaces between numerals.

Graph paper is also helpful in helping pupils learn to properly space two digit numerals.

Peek-a-boo cards

Child is asked to read numeral made with 3 cards stacked - child may peek if can't read the numeral.

MAKE CARDS

100	(100 - 900)
20	(20 - 90 and 11 - 19)
9	(0 - 9)

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

I. (c) The child will read and write symbols for "greater than", "less than" and "equal to".

See Mathmatters pp. 36-40.

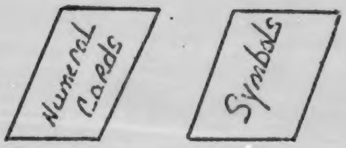
Use a puppet of the greedy duck or greedy crocodile to introduce they symbols. Then use cut out of tagboard or painted pellon to follow up showing the symbols to be a picture of the greedy animal's mouth.

Winner Take All (from Kids Stuff Math)

1. Prepare a deck of cards containing a variety of numerals from 1 - 100
   (at least 15 cards should have a duplicate in the deck.)
2. Prepare a 2nd deck of cards containing one symbol on each card: < > =
3. Deal out all numeral cards
4. Each player, in turn, draws one symbol card and lays down 3 cards that make a true sentence. If she/he cannot make one she/he must pass.

   $\boxed{25}\ \boxed{>}\ \boxed{21}$

5. Players continue until someone is out of cards. Player with most sentences is the winner.



ACTIVITIES FOR LEARNING THE COMPETENCY — MATERIALS

I. (d) The child will read and write the equations for the basic facts with sums to 18 in addition and subtraction (numerals, operation signs,etc

Introduce the signs for plus and minus after children have used concrete objects for developing these concepts (see activities for objectives in IV).

Be sure to call them the PLUS and MINUS signs.

Use 1" tiles with humerals, equality and operations signs printed on with permanent felt mark pen. Have children place tiles in number sentence (equation) forms.

Use dittos from Mathematics Their Way using hands as well as plus or minus sign as pictorial reminder.

WORKJOBS 150-155, 164-165, 168-175,194-211.

FINDERS - KEY-PERS (from Kids Stuff Math)

1. Cut out 19 tagboard "key holes". Label from 0-18.
2. Pin "keyholes" on corkboard that student can carry to working place.
3. Make several keys, each bearing addition or subtraction fact.
4. Place keys in an envelope attached to board. Include several blank keys.
5. Student finds all the keys which could "unlock" the answer in the keyhole and pins them together or hangs on a hook above it.
6. Students may write one problem of their own to add to envelope.

ARIATION: Reproduce keys and keyholes on ditto. Have students cut out keys and glue beneath proper keyholes.

II. (e) The child will begin writing number sentences expressing inequalitie with numbers of nine and less (numerals, operation signs, greater than and less than signs).

See activities for III. (d) - add the greater than and less than signs to the set of 1" tiles and have children make sentences expressing such number ideas as "4 - 1 < 5

In the above example, students should feel comfortable with using manipulatives such as Cuisenaire rods to prove their expressions.)

See Mathmatters pp. 36-40.

See Marcy Cook Count-A-Tile from SET A.

See III. (c) - add problems to numeral cards in winner take all. (2nd grade).

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

II. (f) Pupil will write numerals from 0-999 when shown a manipulative model of the number.

See activity II. (a) for first grade concrete operational and activity II. (e) for second grade. These are good bridging activities to help a child learn to record numerals from the manipulative model of a number.

III. (g) Pupil will write numerals from 0-999 from oral dictation.

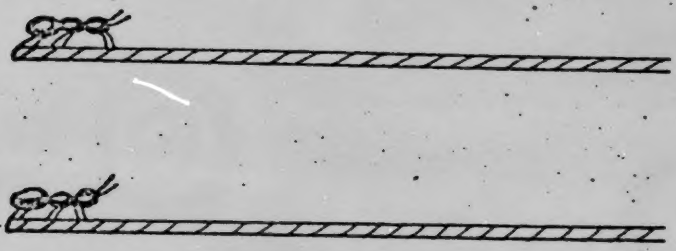
See activity III. (f). Say the number word orally with the child as he records from the model. Then show the child a model of a number using manipulatives on the overhead projector. (Direct child's attention to the screen.) Say the number shown on the screen. Have the child record the numeral. Then ask child to think of the model when you say the number. Challenge the child to write the numeral from the picture in his mind of the model.

Conservation of Length

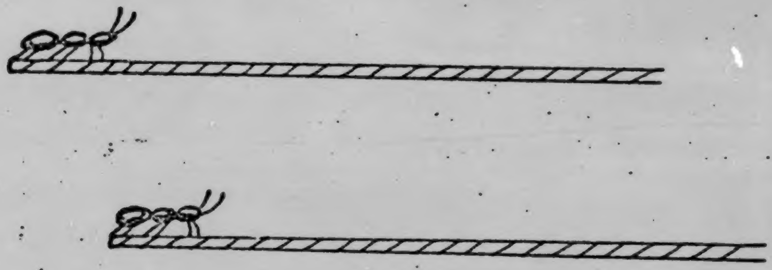
•*Purpose:* This activity will help the teacher evaluate a student's understanding of the invariance (conservation) of the linear qualities of concrete objects when they are moved about.

•*Materials:* Two equally long pieces of heavy cord.

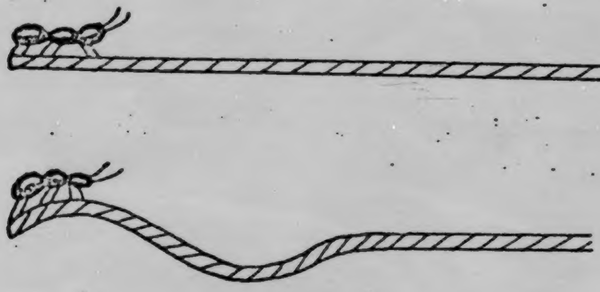
•*Procedure:* Present the following situation to the child: If two equally fast crawly-bugs were put on the end of these strings



and they both went the same speed, would they reach the end at the same time? How about these two strings? (Use the same strings.)

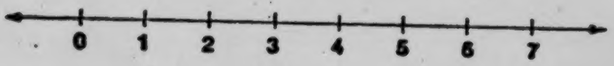


And these? (Again, use the same strings.)



Most children under six years old will not realize that the race would be a tie in the second and third cases even though they know you are using the same string. Around the age of six or seven, most children conserve length and give the correct response because they are able to reverse the process, mentally moving the strings into their original positions.

Children who do not conserve length should not be expected to measure with a ruler, since when they move the ruler it seems to get longer! The number line is of no help to a nonconserver of length. That child perceives the distance between four and five on the number line as being greater than the distance between one and two. Other, more perceptually sound models should be used in such cases.



Souviney, R. *et al*. *Mathmatters*. Goodyear, 1978.

V. (a) Measure objects, using ruler, yardstick, meterstick, and tape measure.

See Preoperational activities for non-standard measures to be used as an introduction as to why standard measures are needed.

Finding a rope for Pero:
Arrange the pieces so than Pero can't get to his food. Make up a reason why the dish cannot be moved. Have the food and flower beds on opposite sides of the dog house with the food several inches or a food closer to the doghouse than the flowers are. Tell the children they need to find a rope long enough for Pero to reach his food but not long enough for him to reach the flower bed because he always tears them up trying to bury his bones. Have the ball of twine fastened to something on the opposite side of the room. Tell them they must go to "the store and buy just the right amount of rope" for Pero. Let the children talk about how to find the right amount. If no one brings up the idea of using a third object (such as an arm, or a measuring evise) to find out how much twine is needed you may have to bring it up. Have them use some standard rule (English or metric) to mark the amount of twine needed and then go "to the store" and see if they can get the right amount.

Large cut outs for bulletin;
large dog
doghouse
dish of food
flowers
twine

(Or display with a toy dog.)

Using Cuisenaire rods for standard measure:
Play a guesing game with a group of children when one person (usually the teacher) holds up an item which can be measured linearly (eg. pencils, book edge, chalkboard eraser) and the children and teacher all take a guess at how many white and orange cuisenaire rods long the item is. Have every one write down their guess. Then let one of the children place the rods on or beside the object to measure it. Have them consider the orange rod as ten so that if they thing there are one orange and three white rods they can write 13 as their guess. (Because the rods are metrically scaled, this also helps the children to internalize the metric system. However, at this point do not discuss centimeters and decimeters - just white and orange rods.)

objects to be measured
Cuisenaire rods

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

IV. (a) cont. Measure objects, using ruler, yardstick, meterstick, and tape measure.

Centers:

Try to keep a measuring center going in the classroom where children can have the fun of measuring everyday objects including themselves and their classmates. Plain meter sticks (nothing smaller calibrated) are good for measuring the large items in the classroom such as walls, chalkboards, bookcases, etc. Tape measures should be available for measuring body parts.

Meter sticks (no calibration)
cloth tapes - met
Yardstick
rulers - 12" and

OPEN MATH RESOURCE BOOK: pp. 14,106-108,229,232-233

WORKJOBS: 218-221

PRIMARY MATH LAB: 26-30

See: Linus Lines from Instructional games section

See Also: Sohns, M.L. and A.V. Buffington.
The Measurement Book. Enrich, 1977.

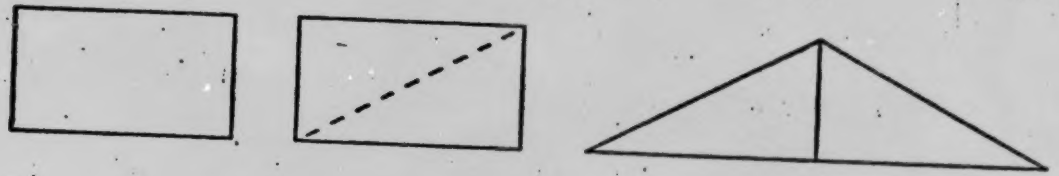
(Fraction Readiness)

Conservation of Area

•*Purpose:* This activity evaluates a student's understanding of the invariance of a concrete object's area when the object is divided into pieces.

•*Materials:* Two equal cardboard rectangles, one cut diagonally to form two equal triangles.

•Procedure: Place the two triangles on the rectangle so that the triangles fit the surface of the rectangle. Once the child has agreed that these areas are equal (i.e. that the same amount of paint would be required to cover the triangles as to cover the rectangle) remove the triangles and arrange them as below.



Ask the child whether the two shapes are still the same size (i.e., whether they would require the same amount of paint to cover them). Most children under the age of seven will say that the second figure is larger. They are guided by their perception: the second figure has more pieces or simply appears larger. Between the ages of seven and eight, most children develop the notion that cutting and moving the pieces about does not change the total area. They are able to mentally reverse the procedure in order to justify their conclusions. Since the definition of fraction is often based on the area concept, formal work with fractions should be delayed until after understanding of area conservation is developed. Introducing informal investigations with fractional parts and fractional parts of sets of objects is appropriate only if conservation-of-number concepts have been mastered.

Souviney, R. et al. Mathmatters. Goodyear, 1978.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
V. (b) Determine area of a rectangle using a geoboard.	
Allow the children to make designs on the geoboards. Then ask if anyone has a rectangle in their design. Ask who has the largest rectangle. Establish the square unit between four nails as the unit of measure. BE SURE TO CALL IT SQUARE UNITS. How many square units does the largest rectangle contain? How many does the smallest rectangle contain? (Note: Some of the children will not realize that a square is a rectangle and will believe that the smallest possible rectangle has an area of 2 square units. This is a good opportunity to clear this up and show that a squ-re is a rectangle because it has four sides opposite sides are parallel and the corners are "square" or 90° angles.)	Geoboards Rubberbands of various colors and sizes
Have the children make rectangles of certain areas which you specify. Everyone make a rectangle which has six square units. Have everyone show their rectangles. Some will be 2x3, others will be 3x2, 1x6, or 6x1. Have them count and see that each ectangle contains the same number of square units - s the same area or size - even though they all look different. Continue the activity with other numbers. Be sure to accept squares as rectangles. What do they notice different aobut seven the twelve unit rectangles. What do they notice different about seven the twelve unit rectangles? (Do not push this but do see if they notice that some areas such as twelve or 16 can be represented by a lot more variation than other such as 5 or 7.)	Geoboards Rubberbands
See Also: Sohns, M.L. and A.V. Buffington. *The Measurement Book.* Enrich, 1977.	
Have children copy a rectangle which you have made. (Use a large geoboard to illustrate or the transparent geoboard on the overhead. Count off the number of nails in each direction.) Ask them to find the area of the rectangle. (Explain that the area is the number of the square units. Be sure they respond that the "area is - squares" or "- square nits." You must work to build the association that rea is always measured in square units.)	

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
V. (c) Measure water into cups, pints, quarts, and liters.	
Allow children to have free time water activities in which they pour the water back and fourth from one container to another. Before structuring any measuring activities make sure they have all had an opportunity to play with the materials and get an intuitive feel for the various sized containers (That quarts and liters are larger than pints and cups, for example, but not the exact relationship.)	Plastic containers (Size clearly marked) cups pints quarts liters Watertable
Ask the children to find out how many cups there are in a quart. Do not direct them in how to find out - let them experiment. At different times ask them such questions as the number of cups in a pint, the number of pints in a quart, how much is a half of a pint, which is larger a quart or a liter, etc.	Plastic containers Watertable
All of the above activities can be done with sand or birdseed instead of or in addition to water.	Sandtable Plastic containers

See Also: VII (a)

See Also: Sohns, M.L. and A.V. Buffington.
The Measurement Book. Enrich, 1977.

Conservation of Mass

•*Purpose:* This activity serves to evaluate a student's understanding of the invariance of the mass of a concrete object when its shape is transformed.

•*Materials:* Two equal amounts of clay.

•*Procedure:* Have the child hold the two pieces of clay and transfer clay from one to the other until both pieces are equal. Roll each piece into a ball and place them on the table in front of the student. Then roll one ball into a long sausage, leaving the other untouched. Ask the child whether both pieces are still the "same amount." Before age eight, most children will answer that the amount of clay has changed. Around the age of nine, however, most children are able to reverse the transformation mentally and to conclude that, since no clay was added or taken away, the mass has remained the same.

The introduction of mass-measurement concepts at the symbolic level to nonconservers of mass is not recommended. Furthermore, models such as the balance, which utilizes the mass concept to define number operations, should be avoided until mass conservation is attained. Considerable concrete experience with such devices, however, will optimize instruction at the primary level.

Souviney, R. et al. Mathmatters. Goodyear, 1978.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

V. (d) Child will weigh self to nearest pound ~~and~~ (if scales available to the nearest kilogram.

If scales available, have children weigh once a week and keep a weekly record. (Include height and other interesting measurements such as chest size.)

A graph showing weights of several children can be made.

ALSO: CHILDREN NEED TO CONTINUE THE BALANCE SCALE ACTIVITIES DESCRIBED IN FIRST GRADE ACTIVITIES.

See Also: Sohns, M.L. and A.V. Buffington. The Measurement Book. Enrich, 1977.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS

IV. (e) Read time to nearest ½ hour.

Use play clocks. Set the clock at different hours. Talk about what we might be doing when the clock shows that time. (eg. 12:00 - we might be asleep if it is night or eating lunch if it is day.)	play clocks

Have the children think of an activity and then set the clock to the hour that they might be doing the activity.

WORKJOBS I 228-229

Have children make play clocks from paper plates, brads, and hands cut from plastic coffee can lids.

See Also: Sohns, M.L. and A.V. Buffington.
The Measurement Book. Enrich, 1977.

IV. (f) MONEY: Identify any combination of coins up to $1.00 and record using both $ with decimal point and ¢.

Play store:

Set up a center with objects priced with amounts from 1¢ to $1.00. Have an older student or an adult work as cashier until the students learn to identify the coins and their value.. Give each student a set amount of play money with which to shop. As the pupils develop skill in identifying coins and counting their own change, allow them to also work as cashier in the play store. (If real objects are used in the store then coins might be earned in the classroom in some manner - or, extra coins might be earned over an above set amount received by each student).

Play money (DLM)
Assorted objects
Price tags
Cash Register

Coin dominoes

These can be purchased commercially or made by the teacher using blank playing cards (or tagboard strips) and rubber stamps of coin faces (see DLM). The object of the game is to match dominoes with the same amount of money shown (eg. a dominoe end with one nickel and five pennies ould match with a domino shownning one dime or with one showing ten pennies.)

Coin dominoes OR
Blank cards
Coin Rubber Stamp

WORKJOBS I pp. 224-227

Make a deck of cards with DLM coin stamps showing various amounts. Make two cards for each amount but using different coins (eg. - 27¢ can be 1 quarter and 2 pennies, 5 nickles and 2 pennies, 1 dime 3 nickles and 2 pennies, 2 dimes and 7 pennies, etc.) Use the cards to play war or concentration. Add one odd card (such as $1.00) and you have an Old Maid deck. Make 4 cards for each amount and you can play GO FISH or RUMMY.

See Also: Sohns, M.L. and A.V. Buffington. The Measurement Book. Enrich, 1977.

Set Inclusion

(Addition and Subtraction Readiness)

•*Purpose:* This activity evaluates a student's understanding of the concept of a whole in relation to its parts.

•*Materials:* Two kinds of flowers (e.g., daisies and carnations)—seven of one (daisies) and three of the other (carnations).

•*Procedure:* Ask the child whether the daisies are flowers and whether the carnations are flowers. Then ask, "If you were to make bouquets, which would be larger, the bouquet of daisies or the bouquet of flowers?" Children under seven will generally indicate that the bouquet of daisies would be larger. They cannot simultaneously consider the *whole* set of flowers and at the same time distinguish between the *parts* (daisies and carnations). The child who is unable to deal with set inclusion is unable to reverse the process by breaking up a whole and realizing that a recombination of the parts is once again equivalant to the whole. This is analogous to a child's knowing that $3 + 5 = 8$ but not recognizing that $8 = 3 + 5$. Around the age of seven, children are able to consider both the whole and its parts simultaneously and can therefore solve the set-inclusion problem.

Before children have acquired a thorough understanding of set inclusion, it is not advisable to introduce the operation of addition and subtraction at the formal (symbolic) level. When addition is introduced before a child has gained competence with set inclusion, any learning may be rote in nature and true understanding of the concept may be delayed.

Souviney, R. et al. Mathmatters. Goodyear, 1978.

Transitivity

(Multiplication Readiness)

•*Purpose:* This activity evaluates a student's ability to recognize the equivalence of sets in an array rather than depending on one-to-one correspondence or counting methods to make that determination.

•*Materials:* Ten daisies, ten carnations, and ten vases.

•*Procedure:* Have the child put one daisy in each vase. Remove the daisies and repeat the procedure with the carnations. Ask whether he has used the same number of daisies as carnations. Then ask, "If all the flowers were put back in vases in the same manner as before, how many flowers would be in each vase?"

Children under six years old are generally unable to answer either question correctly. They are unable to conserve number, so when the daisies are removed and replaced by the carnations, they must re-count or place them in one-to-one correspondence in order to justify equality. These children are also unable to use the transitive property to assist them in solving the problem. That property could be described as follows:

Number of Daisies = Number of Vases

Number of Carnations = Number of Vases

Number of Daisies = Number of Carnations

Between six and seven years of age, however, children generally solve both problems readily, providing correct responses to the second question even when three, four, or more sets of flowers are considered.

Competence with the transitive property is of primary importance in the learning of multiplication skills. Transitivity and set-inclusion concepts develop in children at approximately the same time. This implies that while in the mathematics programs presently in use addition may be introduced a year early, introduction of multiplication may take place a year late! In any case, systematic introduction of symbolic multiplication should be delayed until after competence with transitivity is attained.

Souviney, R. et al. Mathmatters. Goodyear, 1978.

THE ASSOCIATIVE PROPERTY OF ADDITION

> **Unlike seriation and transitivity, associativity involves structures that cannot easily be fixed in the memory.[11]**

Purpose of Activity

In this activity children's understanding of, and memory of, the associative property of addition will be explored.

Materials Needed

1. Two sets of counters—one of 8 and one of 14.
2. Four glasses and some liquid, as illustrated on the following page.

Procedure

In Activity 2-10 on addition and subtraction, the child was asked to equalize sets of 8 and 14 counters. One way of doing this could involve thinking of the 14 renamed as 8 + 6 and dividing the 6 left over between the two sets. Expressed numerically,

$$\begin{aligned} 8 + 14 &= \\ 8 + (6 + 8) &= \\ 8 + (3 + 3 + 8) &= \\ (8 + 3) + (3 + 8) &= \\ 11 + 11 &= \end{aligned}$$

The whole amount in the two sets 8 + 14 is not changed by regrouping the set of 14 from 6 + 8 to 3 + 3 + 8 and associating one of the addends, 3, with the addend, 8.

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[11] Jean Piaget and Barbel Inhelder, *Memory and Intelligence* (New York: Basic Books, Inc., 1973), p. 117.

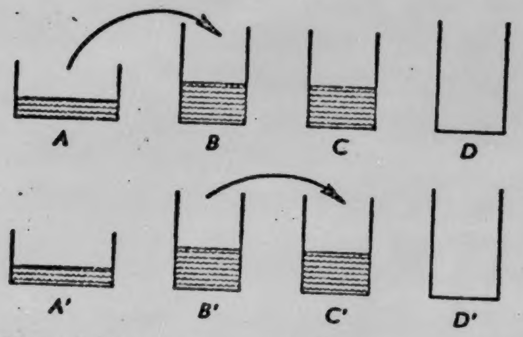
Does it make any difference, in adding any three or more numbers, which pairs of numbers you group or add first? For example, does

$$\begin{aligned}(3 + 2) + 5 &= 3 + (2 + 5)?\\ 5 + 5 &= 3 + 7\\ 10 &= 10\end{aligned}$$

The parentheses indicate which number pairs are associated first, and it is seen that the different associations produce the same sum. Since this is true for any whole numbers a, b, and c, the idea is called **the associative property** and is usually stated as

$$(a + b) + c = a + (b + c)$$

To explore this idea at the physical or concrete level, Piaget used a process of pouring liquids.



The child is shown the display as illustrated, and is told that each glass holds the same amount as the one above (or below) it. The interviewer then pours the liquids as indicated by the arrows.

Will you get the same amount if the liquids are grouped differently; that is, do you get the same amount by pouring (indicate by pointing, not naming letters) A into B and the result into C as by pouring A' into B' and C' combined? The child is then asked if these (B and C) are poured here (pointing at D), and if these (pointing at A' and C') are poured in here (pointing at D'), will you get the same or more in one than in the other? If the child thinks there will be more in one, ask which one and why. Symbolically, this can be represented as

$$(A + B) + C = A' + (B' + C')$$

The interviewer then pours the liquid into D and D', so that the child can see that they do contain the same amount, and the child is asked if she can explain why.

If the interviewer is interested in how this activity relates to memory, the child is asked one week later if she can remember what she did and if so, to draw it. The materials are then brought out and the child asked to do what was done the week before. Finally, she is asked if it is the same here and here (meaning D and D'). The same procedure is repeated six months later to see if, in the interim, the child has acquired better cognitive processes to resolve the problem. (For a full discussion see Piaget and Inhelder's *Memory and Intelligence,* Chapter 6.)

Levels of Performance

Piaget finds five levels of performance as far as memory is concerned. Because the description of these levels is so lengthy, it will not be attempted here. (See *Memory and Intelligence,* pages 118–23.)

Children's understanding of conservation of quantity seems to be a prerequisite to understanding associativity. The child must understand that if the liquids are the same in A as in A', in B as in B', and in C as in C', then pouring them into different-shaped containers does not change the amount.

At 7 to 8 years of age, children do have a stable concept of conservation of quantity; thus, they are also able to remember some of the associations made (how the liquids were poured).

At 8 to 9, some of the children are able to use the idea of the associative property (that the different associations make no difference and that, therefore, there will be the same amount in D as in D').

But Piaget points out that in the case of associativity, subjects 6 to 9 years lack adequate schemata, having only conservation schemata. Differentiated associativity does not emerge until later, and is not fully consolidated until 11 to 12 years of age.[12]

Teaching Implications

The associative property is more difficult for children than many elementary math textbook writers realize. Some first-grade books include this idea. It would seem more appropriate to delay introducing the idea until the child reaches the second or third grade, realizing as described in the preceding paragraph that a full understanding of associativity does not occur until 11 to 12 years of age.

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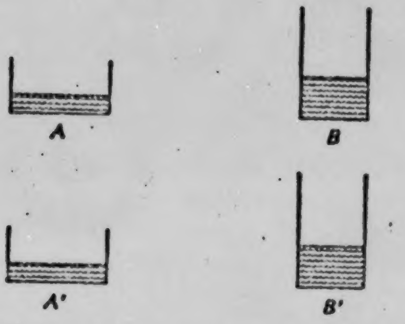
[12] Ibid., p. 126.

A further study of associativity can be made by means of Activity 2-10 or Piaget's *The Child's Conception of Number,* page 190.

While the apparatus for this activity is set up, it may also be interesting to investigate the commutative property of addition; that is, does the order of adding liquids affect the amount?



If *A* and *A'* contain the same and *B* and *B'* contain the same, do we get the same amount pouring *A* into *B* as *B'* into *A'*? (In conducting the activity, do not use letters but point at the appropriate container.)

Copeland, R. W. Math Activities for Children. Merrill, 1979.

ACTIVITIES FOR LEARNING THE COMPETENCY

MATERIALS

. (a) Use real objects to add and subtract numbers with sums to 18.

NOTE TO TEACHER: One reason children have so much trouble with subtraction in later grades is that they learn only the "take-away" model for subtraction in the earlier grades. In real life (and in story problems) subtraction is almost always the "comparison" model and is more often the "missing addend" model than it is the "take away" model. PLEASE be sure that you use examples of more than one model when teaching the concept. DO NOT let the children confuse TAKE AWAY WITH SUBTRACTION.

Take away model activities:

Use real objects to divide sets into subsets; remove one subset; verify the number remaining. The children may count at the beginning to find the number remaining. Follow by giving several different problems in a row (using different objects) which use the same numbers (eg. 7 - 3 = 4). After two or three of these problems, ask the children to predict before counting the amount that will be left the next time. "Do you suppose that when we take 3 from 7 there are always four left? Why?" This leads to the idea that if the answer if always going to be the same for certain combinations we can just remember the answer instead of having to count out objects each time.

Numerous sets of objects

(eg. blocks
buttons
paper clips
pencils
crayons
dishes
dolls
toys
books, etc.)

Comparison model:

The Cuisenaire Rods are an ideal material for introducing this model of

V. (a) cont. Use real objects to add and subtract numbers with sums to 18.

subtraction. Children find THE DIFFERENCE between the yellow rod and the brown rod by laying the yellow rod on top of the brown rod and finding that the light green rod will fill up the amount of space that is left.- or the difference. They can also fill up the amount of space with three white rods. Given the rule white = 1, then, they can discuss the idea that the difference between 5 (yellow) and 8 (brown) is 3 (light green).

Centimeter cubes, joined together, can be used as an example of this model. Make two rods of different lengths from the cubes. Have the child compare the rods and determine which is larger. "How many more cubes does this rod have? What is the difference between the two rods?" This can be a bridging activity from the take-away model, since the child can take the extra cubes from the longer rod before counting them.

Missing Addend Model:

A pan-balance and centimeter cubes (or other items of a uniform mass) can be used to model the missing addend. Place five cubes on one side of the balance and two cubes on the other. Ask "How many cubes do I need to add to the two cubes to make the pans balance?" Let the children add them one at a time if they cannot predict the correct amount. "Two plus how many was five?"

A number balance is also an excellent model for this, however, most first graders seem to think of it as a magic trick. It is better to use it at later grades.

V. (a) cont. Use real objects to add and subtract numbers with sums to 18.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
Place a number of dissimilar objects on a table (dissimilar so that children will not develop the idea that the members of a set must have some characteristic in common other than that they are members of the same set). Ask children to name the members of the set (eg. a block, a pencil, a key) as each is pointed to. The set will be well defined if the children can answer such questions as these: "Is the block a member of the set?" (yes) "Why?" (It's there.) "Is the elephant a member of the set?" (no) "Why?" (It's not there.)	several objects (one of each) (eg. pencil block key eraser, etc.)
Use the flannel board to develop the idea of joining sets to find the number of a new set and begin work on the commutative property of addition. Ask the children, "What things do you see in each set?" How many members are in the first set? How many in the other set?" Move the set with an apple and duck (for example) to join the other set (eg. bunny, star, pear) and ask, "If you join a set of two with a set of three, how many will be in the new set?" Separate the sets and then join the set of three with the set of two and ask the same question. Present several different joinings, using numbers other than zero. The children may need to count to determine the number of the new set. After this routine has been established, use different objects but the same number of objects three or four times in a row, (eg. 4 and 3). After these numbers have been used several times, take out a new set of 3 and a new set of 4 and ask the children to predict how many members the new set will have after they are joined. "Do you suppose 3	Flannel board · Felt pieces of many sizes and shapes

V. (a) cont. Use real objects to add and subtract numbers with sums to 18.

objects and 4 objects will always join to make 7 objects?" Let them try it many more times if they are not sure. This is an important step because it establishes the idea that if the sums are always the same we can just remember the answer instead of counting out objects each time we need to know the total of 4 and 3. (The above activities may be done with a smaller group using real objects on trays or tables instead of the flannel board.)

WORKJOBS 150-151, 174-175, 194-211

WORKJOBS II - all activities

THE COMMUTATIVE PROPERTY OF ADDITION

Purpose of Activity

What is children's understanding of the commutative property of addition—that the order of adding numbers does not change the sum? For example, when do children realize that 2 + 7 = 7 + 2?

Materials Needed

Two sets of objects, such as 2 yellow toy cars and 7 blue cars.

Procedure

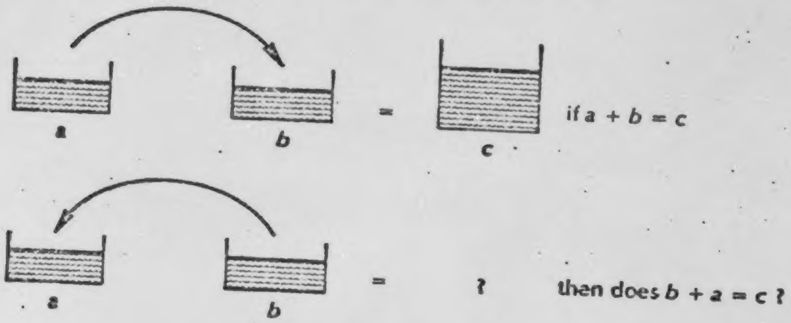
Ask the child to line the cars up in a row with all the cars of one color followed by all the cars of the other color. Put a parking marker at both ends of the row. Then remove the cars from the row. If the

Checking to see if a row of two "Mickey Mouse" dolls followed by three dwarfs is the same length if the order is reversed.

child began his row with yellow cars, call that to her attention and ask her to do it again beginning with the blue cars. After she has placed two or three cars, stop her and ask, "Do you think this row will reach to the same parking marker, or not as far, or farther? Why?"

A variety of other materials could be used for the same purpose—dolls of two types, for example.

Also, to investigate the idea further, use continuous sets, such as liquid to which food coloring has been added.



Levels of Performance

Children will be successful with these tasks on the average at 7 to 8 years of age. The tasks involve the concept of reversibility which develops around 7 years of age.

Teaching Implications

Present practice of teaching addition in the first grade ignores the necessity of reversible thought as a basis for understanding the operation. It is one thing to teach that 3 + 2 is equal to 5 and that 2 + 3 is equal to 5 and have children memorize that the response is 5 in both cases; but the activity just presented demonstrates that teaching the concept is not the same as teaching a response.

When children *do* have reversible thought, the commutative property simplifies teaching the 100 basic addition facts, because for every addition fact such as 7 + 2, they also know 2 + 7.

Copeland, R. W. Math Activities for Children. Merrill, 1979.

. (b) Recall addition facts through sums of 18.

See activities for V. (b) in first grade concrete operational curriculum.

Make decks of flash cards for each addition fact. Have children test each other in pairs. Have the child doing the testing hold up the card and count "1001, 1002, 1003" under her/his breath. If the child being tested gives the correct answer in this time limit, that child keeps the flash card. If the child cannot answer, the tester keeps the flash card.

At the end of the activity, the children give the cards to you and identify which stack were known and which stack were unknown. Make a ditto master of the addition fact chart with a place for the students name. Make enough copies for you to have one for each child. When you receive the cards from the tester, take the ditto sheet for the student who was tested and circle every fact missed. (You hould soon have a record of which facts every child n your class knows or does not know.) Analyze the child's sheet. Does she/he understand the commutative property or does she/he answer 7+8 correctly and miss 8+7? These charts can be used for individual conferences with students where you both agree on certain facts to be learned each week. They can also be used when grouping children for drill and practice learning centers and games (children who need the same practice or by having a child who can check with one who has not yet mastered all facts being used in the activity.)

. (c) Recall subtraction facts through sums of 18.

See activities V. (c) from first grade concrete operational curriculum.

See diagnostic and record keeping procedure outlined in V. (b) of second grade curriculum and adapt to subtraction facts.

MULTIPLICATION OF NUMBERS

Purpose of Activity

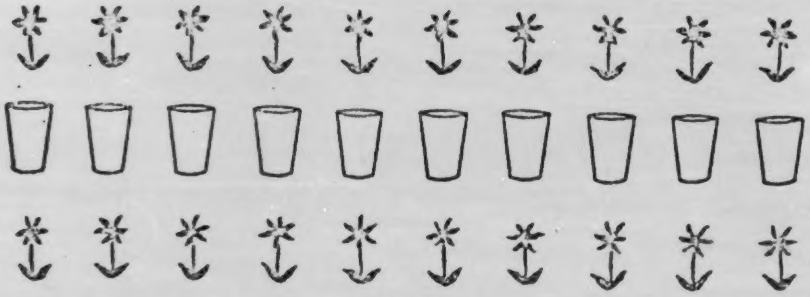
The purpose of this activity is to determine when children are able to solve problems involving multiplication of numbers.

The ability of children to make a matching or one-to-one correspondence between sets was studied in Activity 2-6 on conservation of number. From the psychological standpoint, a one-to-one correspondence is an implicit multiplication, for if two sets are in one-to-one correspondence, then the total number is twice or two times the number in each set.[13]

In this activity more than two sets will be considered in order to study children's ability to predict the total number as two, three, or four times the number in each set.

Materials Needed

Cardboard cutouts of 10 blue flowers, 10 yellow flowers, 10 vases.



Procedure

Ask the child to put a blue flower in each vase. Then remove the flowers and ask her to put a yellow flower in each vase. After these are removed, ask her if there are the same number of blue flowers as yellow flowers. And then, "If we put all the flowers in the vases with the same number in each vase, how many would there be in each vase?"

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[13] Jean Piaget, *The Child's Conception of Number* (New York: W. W. Norton & Company, Inc., 1965), p. 204.

Levels of Performance

Stages 1 and 2. Between 5 and 6 years of age, the child can make a one-to-one correspondence of flowers to vases, but if one set of flowers is bunched up afterwards and the other set spread out, she thinks there are more flowers in the spread out set. She does not have the conservation of number concept.

Also, the child does not have the transitive idea:

If the number of

(blue flowers) = (vases)

and

(yellow flowers) = (vases)

then

(blue flowers) = (yellow flowers)

This transitive property of the equals relation may be expressed symbolically as

if $a = b$ and $b = c$, then $a = c$

The Stage 1 child cannot apply this logical idea.

At Stage 2, the child gradually succeeds by repeating the experiment or through the interviewer's questions such as "How many blue flowers were in the vases?" and "How many yellow flowers were in the vases?" But perceptual factors still influence her—as one set is spread out more, for example.

Stage 3. At 6 to 7 years of age, the child can immediately use the logic of the transitive property and conclude that since one blue flower went in each vase and one yellow flower went in each of the same vases, there must be the same number of blue flowers as yellow flowers.

The Stage 3 child can also answer the second question successfully: "If we put all the flowers in the vases with the same number in each vase, how many flowers are in each vase?"

If a third set of ten flowers is introduced and the question repeated, she can predict that each vase will hold three flowers.

If a set of straws is introduced with the restriction that only one flower will go in each straw, and the child is asked, "How many straws will we need for all the flowers?" she can predict how many. The Stage 2 child would have to go through the process of pairing straws and flowers to find the answer and then realize why the answer is what it is.

Teaching Implications

Multiplicative operations are seen to develop at the same time as additive operations in the mind of the child. Yet multiplication is not studied in comprehensive fashion until the third or fourth grades in most schools. This is probably because there are 100 addition facts to learn, and although they may not be easier, it takes time to learn them. Part of the problem may be beginning addition too soon and multiplication too late.

Copeland, R. W. Math Activities for Children. Merrill, 1979.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

I. (a) Use real objects to add and subtract multidigit numbers.

See the following pages from Open Math Resource Book illustrating multi-digit addition and subtraction with manipulative models. Do not use numerals when first introducing procedure.

NOTE: THE CHILD MUST DO THE MANIPULATING. THESE ARE NOT TO BE TEACHER DEMONSTRATIONS.

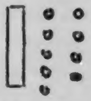
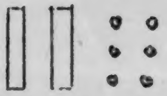
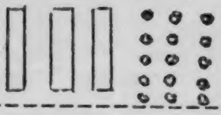
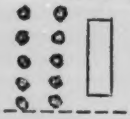
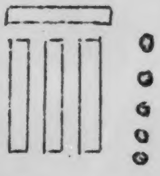
INTRODUCE:

Try to use real situation such as there are 19 boys and girls present today in our room and 24 boys and girls in room x. If we all go to P.E. together how many boys and girls will be in P.E. Have students model the 19 and then model the 26. Have them join the sets. Then ask if they need to rebundle or trade (depending on material) to make the number easier to "read." Find regrouped number is 45.

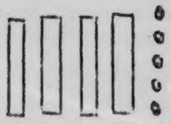
After several informal situations (try to use real problems that involve regrouping right from the beginning) introduce more formal situations using a mat such as shown in Open Math or Mathmatters. (Mats can be made of butcher paper but pellon is very durable and can be kept for years.)

VI. (b) Record multi-digit addition and subtraction problems from a manipulative model.

After several experiences such as those described in VI. (a), tell the children you are going to record what they are doing at the chalk board as they do it. For most children you would record directly into numerals as demonstrated in Open Math or below. For some children the pictorial model suggested by Mathmatters may be a necessary requirement. (Often a pictorial can be saved for use if a child has difficulty going directly from concrete to abstract.)

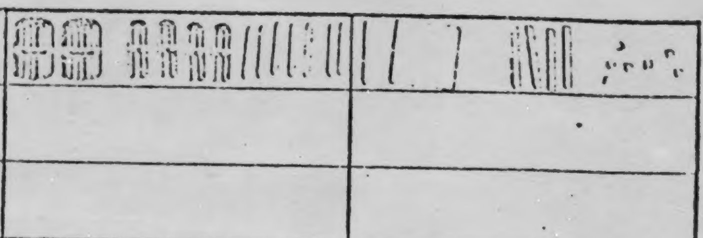
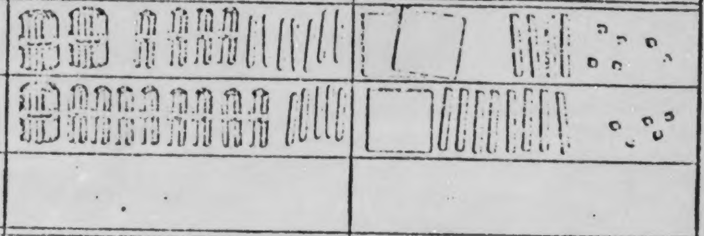
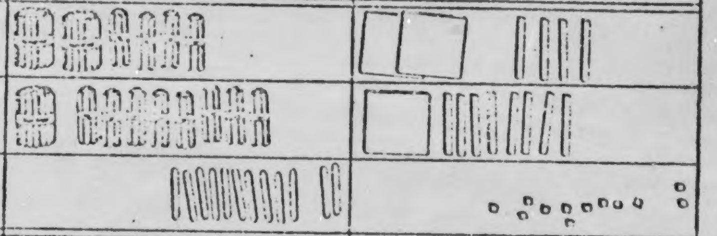
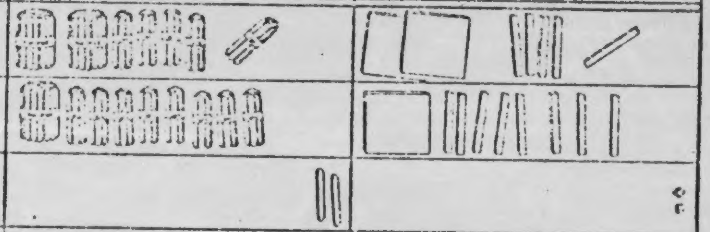
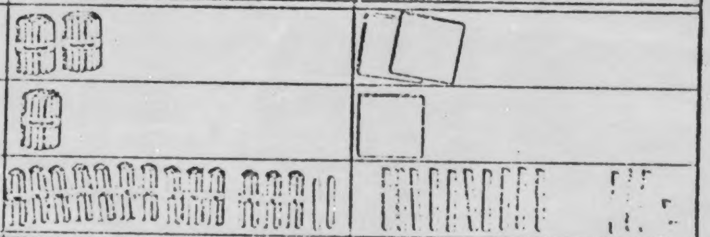
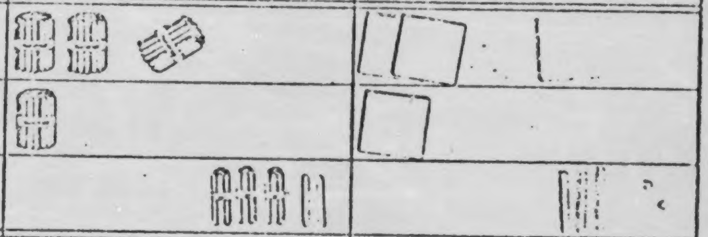
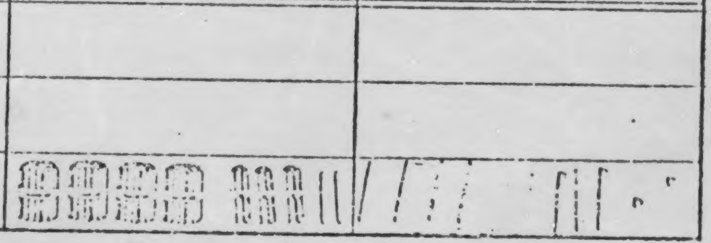
Teacher Says	Child Does	Teacher Writes
"Represent the 19 boys and girls"		19
Represent the 26 boys and girls		19 26
"Now join the two sets together"		19 +26
-OPTIONAL- (Teacher can write this or omit intermediate algorithm - depends on the group of children)	→	19 +26 15 30
"Do you need to do any regrouping (or trading)?"		
(If not using optional algorithm) "Let's see - how will I record what you have done? I need to show that you traded 10 units for a 10." "I will made a one in the 10's column to show the trade."		1 19 +26 5

VII (b) Cont. Record multi-digit addition and subtraction problems from a manipulative model.

Teacher Says	Child Does	Teacher Writes
"Now what was your total?"		$\begin{array}{r}19\\+26\\\hline 45\end{array}$

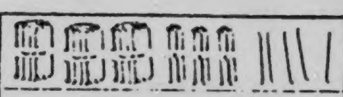
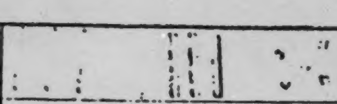
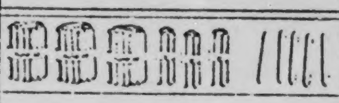
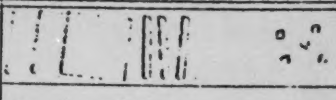
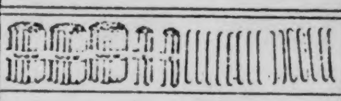
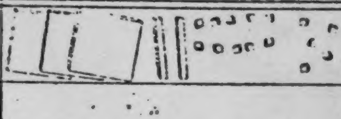
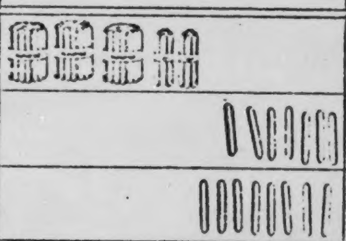
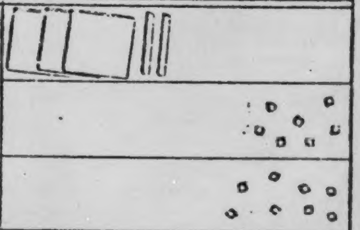
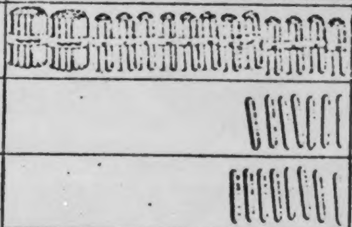
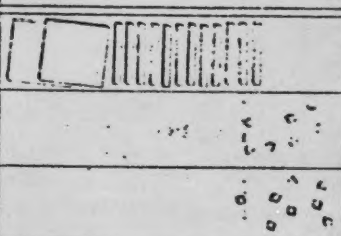
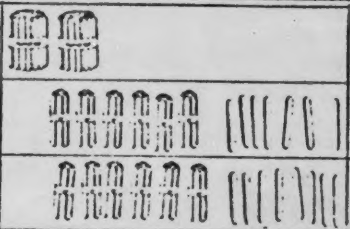
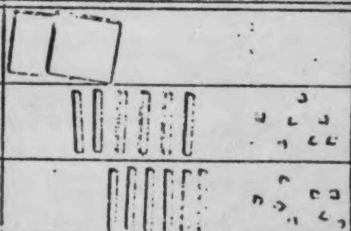
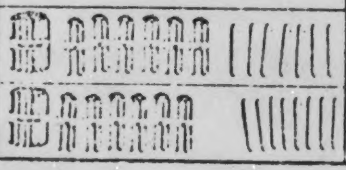
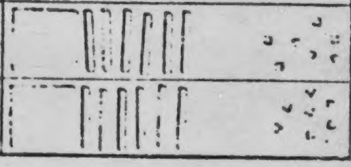
After 2 or 3 times of teacher recording from child's model, child should record from his/her own model.

ADDITION PROCEDURE

Represent the first addend.	247 +	
Represent the second addend.	247 +185	
Combine the ONEs; form a group of ten and a group of two	247 +185 2	
Trade the ten ONEs for one TEN.	1 247 +185 2	
Combine the TENs; form a group of ten and a group of three.	1 247 +185 32	
Trade the ten TENs for one HUNDRED.	1 1 247 +185 32	
Combine the HUNDREDS.	1 1 247 +185 432	

Knaupp, J. Open Math Resource Boo

SUBTRACTION PROCEDURE

Represent the minuend.	335 -		
Determine what the subtrahend will be.	335 - 167		
There are only five ONEs, so a TEN must be exchanged for ten ONEs. Now there are fifteen ONEs.	$3\overset{2}{\cancel{3}}{}^{1}5$ - 167		
Take away seven of the fifteen ONEs, leaving eight.	$3\overset{2}{\cancel{3}}{}^{1}5$ - 167 8		
There are now only two TENs, so a HUNDRED must be exchanged for ten TENs. Now there are twelve TENs.	$\overset{2}{\cancel{3}}\overset{12}{\cancel{3}}{}^{1}5$ - 167 8		
Take away six TENs, leaving six TENs.	$\overset{2}{\cancel{3}}\overset{12}{\cancel{3}}{}^{1}5$ - 167 68		
Take away one HUNDRED, leaving one HUNDRED.	$\overset{2}{\cancel{3}}\overset{12}{\cancel{3}}{}^{1}5$ - 167 168		

Knaupp, J. Open Math Resource Bo

Because the concept of place value is lost when the two digits are thought of as separate entities, children understandably encounter difficulty when they must eventually learn to regroup. Suddenly, the two digits must be related in some manner and cannot be dealt with separately.

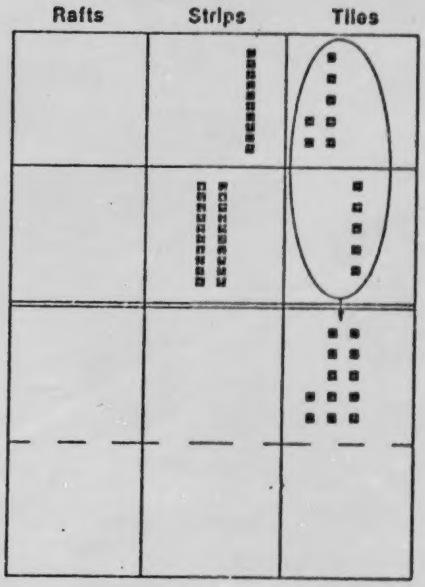
To avoid this confusion, two-digit addition with regrouping must be introduced simultaneously with nonregrouping situations. The concept should be presented in such a manner that both facets (regrouping and nonregrouping) are viewed as a natural and integral part of the process of addition.

Activity 2-G1.
(concrete)

•*Materials* Tiles, tile strips, Work Sheet 2-G1.

•***Procedure*** Students are given boards and asked to work problems that are given to them verbally. Again, place values are represented on the board by columns for single tiles, strips of ten, and rafts of one hundred.

When asked, for example, to add seventeen and twenty-five, students first combine all the single tiles (ones) and move them to the answer block.



Students are then asked how many individual tiles there are in all and whether this is the "best way" (see Activity 2-B1) to show twelve. They should respond that the best way would be by means of a tile strip and two single tiles. At this, the teacher points out that students must therefore trade ten individual tiles for a tile strip and move the tile strip into the appropriate column.

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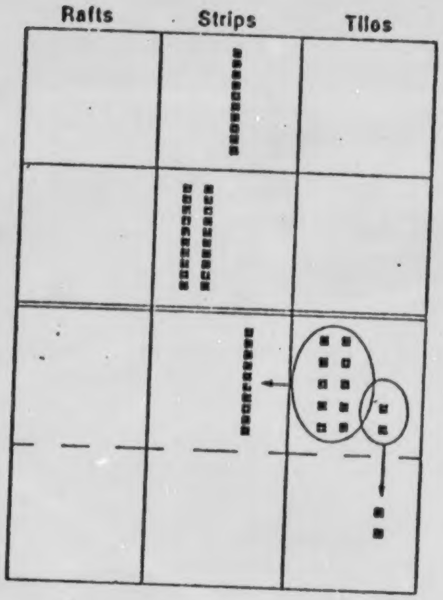
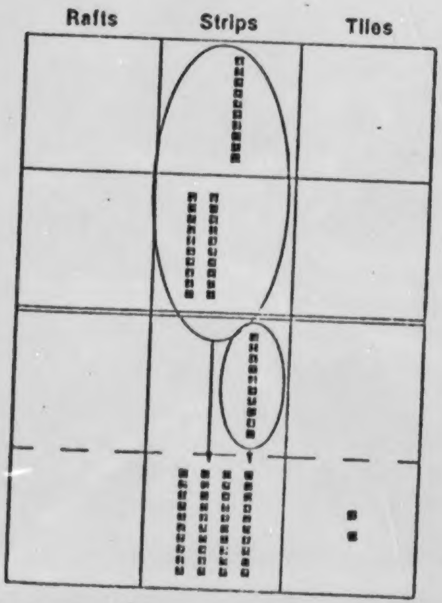

Rafts
Strips
Tiles

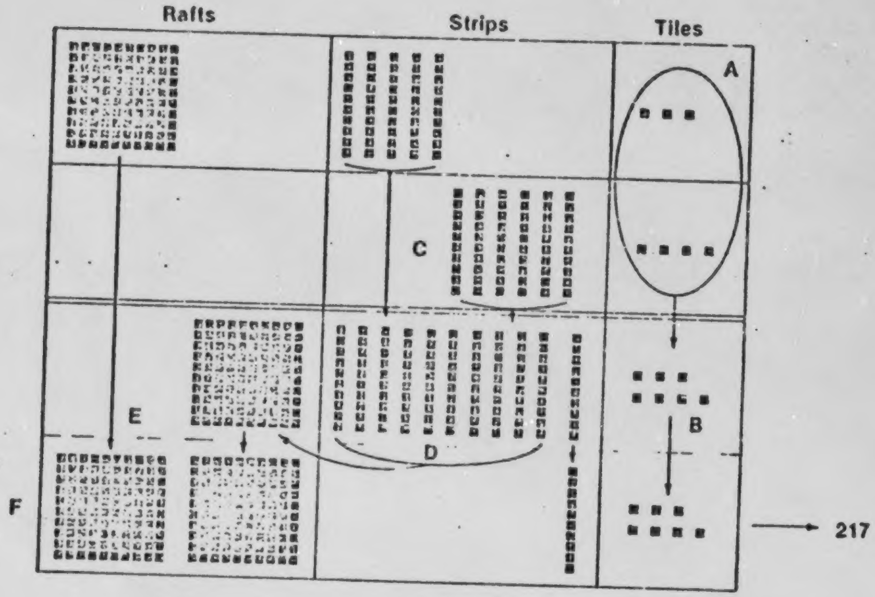
The same process is then repeated for the tile strip column.


Rafts
Strips
Tiles

Souviney, R. et al. Mathmatters. Goodyear, 1978.

The resulting group of tile strips (including the one compiled from the tiles place) is examined to see whether it, too, is the "best way" to show forty. Since in this case forty is already represented correctly, the problem is completed and the answer is forty-two.

Here is how the sum 153 + 64 would be handled:



All the single tiles are combined (A) and then checked to see whether the sum is represented in the "best way" (B). in this instance, it is represented in the best way, so we leave it alone. Next all the tile strips are combined (C). This time we must change the tile-strip group to the "best way" (D) by trading ten strips for a raft of one hundred. The raft is placed in its column (E) and with the other raft is brought down to the answer block where they are again checked for the "best way." Thus, the answer (F) is obtained.

Activity 2-G2.
(concrete/ representational)

•*Materials* Tiles, tile strips, addition organizer, pencils, Record Sheet 2-G2.

•*Procedure* Once again, give students an addition problem to be solved by means of addition boards, tiles, and tile strips. Ask them to use our representational notation in order to record the problem-solving process. In this way they experience both a concrete situation,

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Name____

ADDITION ORGANIZER

	Tile Rafts	Tile Strips	Tiles
+			
	___ ___ ___	___ ___ ___	___ ___ ___

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Name____

1.

Rafts	Strips	Tiles

2.

Rafts	Strips	Tiles

3.

Rafts	Strips	Tiles

4.

Rafts	Strips	Tiles

5.

Rafts	Strips	Tiles

6.

Rafts	Strips	Tiles

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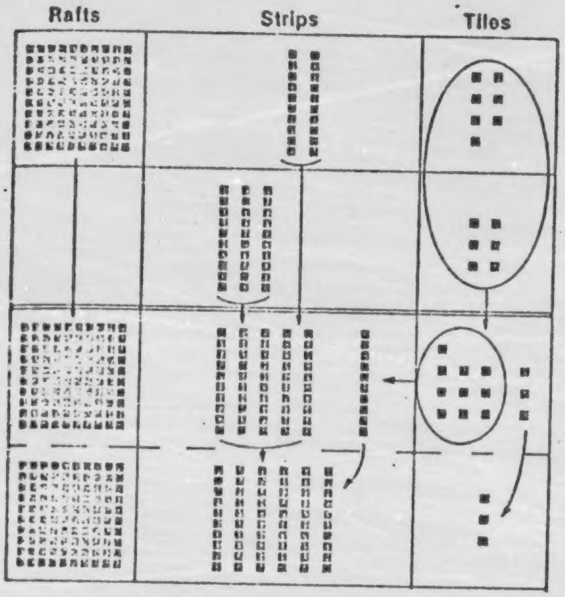
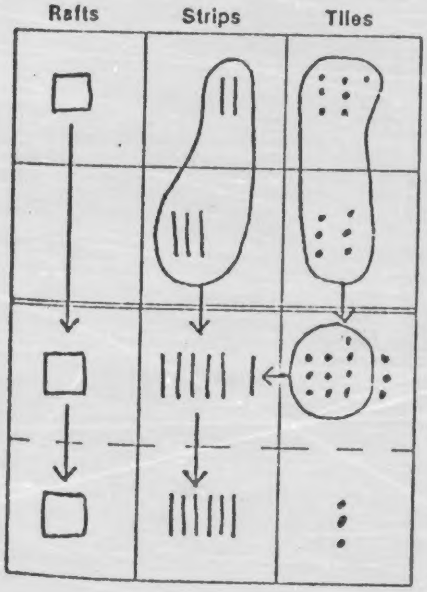

Rafts
Strips
Tiles

127
+ 36

(Numbers are for teacher reference only.)

and the making of a representational record.


Rafts
Strips
Tiles

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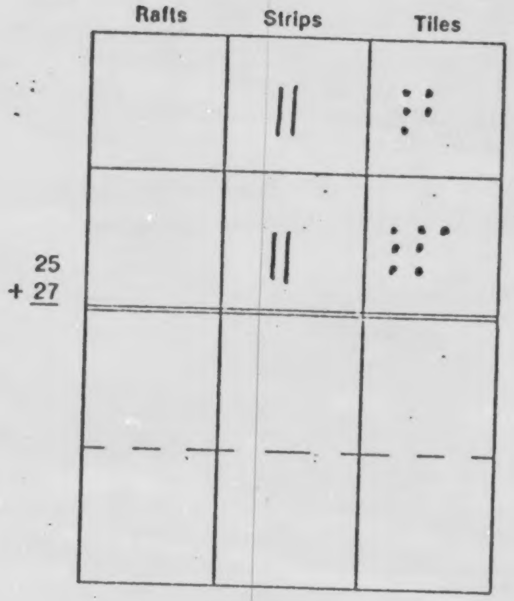
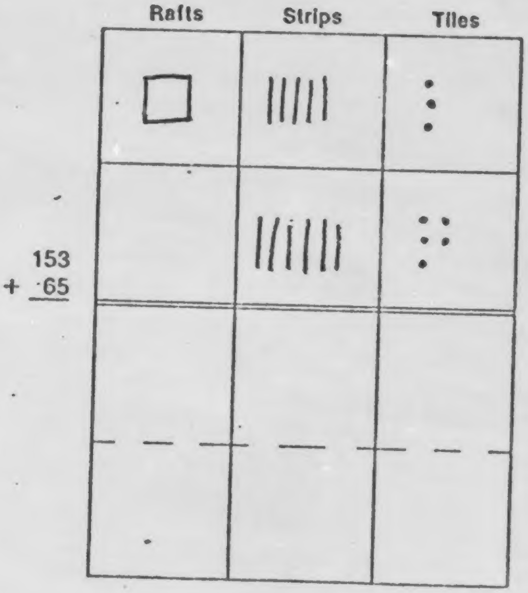
In contrast to the concrete model, representational notation allows the original addends to remain visible. After students have acquired extensive experience in using this notation as a recording process, the tile-strip model may be eliminated and all work done at the representational level.

Work sheets like the following may be developed (numerals are for teacher reference only; problems are given orally):





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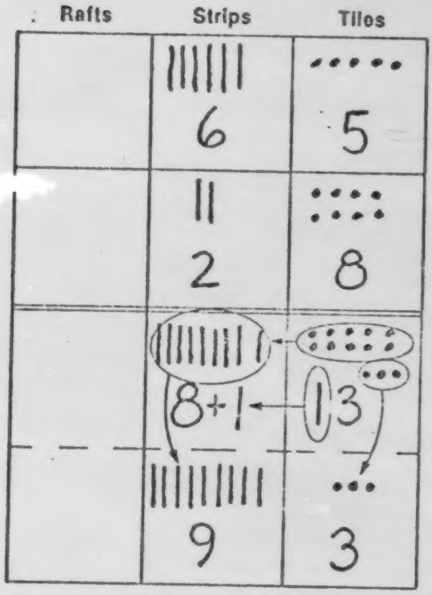
Activity 2-G3.
(representational/ symbolic)

•Materials Pencils, tiles, tile strips, Work sheets 2-G2 and 2-G3.

•Procedure Students are given representational problems and asked to make a symbolic record of their transactions on Record Sheet 2-G2.

When students have become familiar with the mechanics of symbolic notation, the representational schema is eliminated and problems are handled in an entirely symbolic manner (Work Sheet 2-G3). After computing the answer, students should write the entire problem next to the chart.



100s	10s	1s
	2	6
	9	2
	11	8
1	1	8

26
+ 92
118

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ADDITION

Name____

Example:

A.

	Hundreds	Tens	Ones
		3	4
+		2	8
		5	12
		6	2

$$\begin{array}{r}34\\+\underline{28}\\62\end{array}$$

1.

	Hundreds	Tens	Ones
	1	2	6
+		9	2

2.

	Hundreds	Tens	Ones
	1	2	7
+	2	1	1

3.

	Hundreds	Tens	Ones
	5	2	9
+		7	8

4.

	Hundreds	Tens	Ones
	2	9	0
+	3	7	8

5.

	Hundreds	Tens	Ones
	7	0	6
+		9	8

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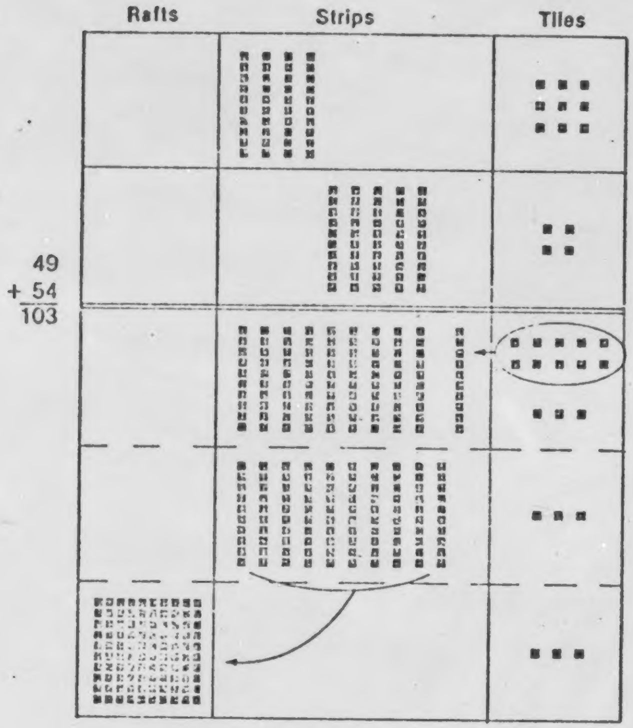
Ask them to validate these symbolic manipulations using tiles and tile strips.



Students first complete the problems at the symbolic level, then validate their answers using concrete materials.

Activity 2-G4.
(symbolic)

•*Materials* Pencils, Work Sheet 2-G3.

•*Procedure* A fascinating phenomenon for children to explore, as well as a welcome alternative to the traditionally dull routine of drill and practice, is the palindrome, a word or number that reads the same forward and backward: pop, tot, toot, radar, 101, 2112, 23232, 111. Palindromes can also be complete sentences: Madam, I'm Adam.

A most interesting aspect of the numerical palindrome is the way in which it can be derived. Pick any number, reverse it, and add the two.

$$\begin{array}{r} 421 \\ +\ 124 \\ \hline 545 \end{array}$$

The result is a palindrome!

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Sometimes it takes more than one step.

273	
+ 372	
645	
+ 546	Reverse and Add
1191	
+ 1911	Reverse and Add
3102	
+ 2013	Reverse and Add
5115	Palindrome

This will always work. You can use any number and eventually you will reach a palindrome! A calculator might be handy if you try the number 89.

Concept H: Subtraction with Regrouping

Note: When students are working with multiple-digit numbers, subtraction with and without regrouping are introduced simultaneously as two facets of the same concept that should not be separated (see the introduction to Activity 2-G1).

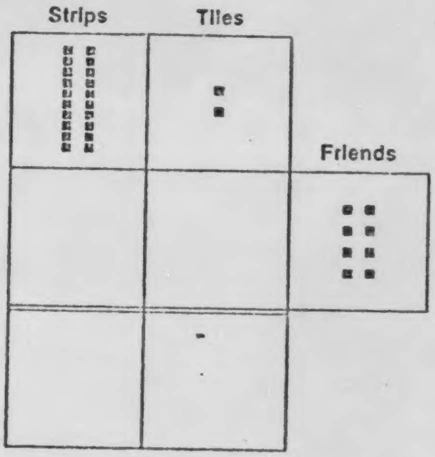
Activity 2-H1.
(concrete)

- *Materials* Tiles, tile strips, subtraction boards.
- *Procedure* Subtraction problems are set up in this manner:

Example 1.

(Numbers are for teacher reference only.)

$$\begin{array}{r} 22 \\ -\underline{\ \ 8} \end{array}$$



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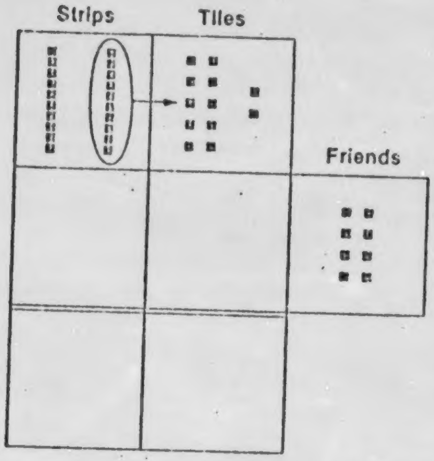
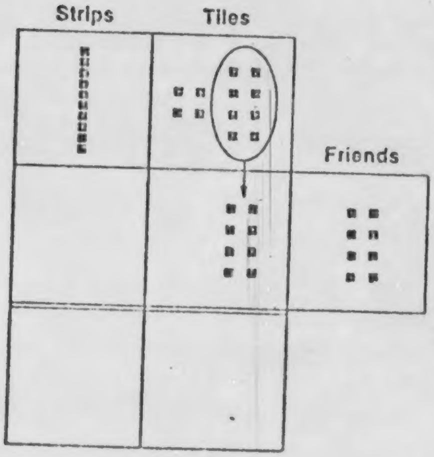
The tiles in the far-right box represent the "friends" used when developing the concept of subtraction (see Activity 2-F2). This should be made evident to the students.

Ask students whether they have enough single tiles to give each friend one. After they respond negatively, ask them where they could possibly get more. Encourage them to exchange a tile strip for ten individual tiles, which are then placed in the appropriate position.



Now are there enough tiles for each friend to have one?

The process may now be completed.



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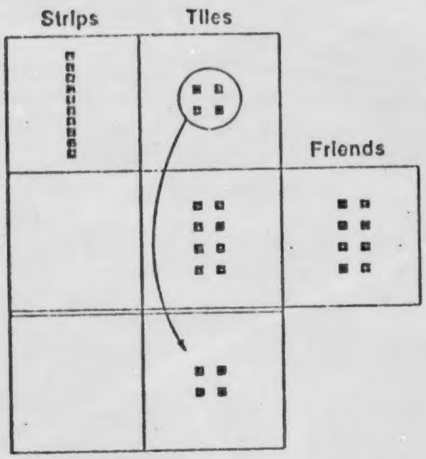
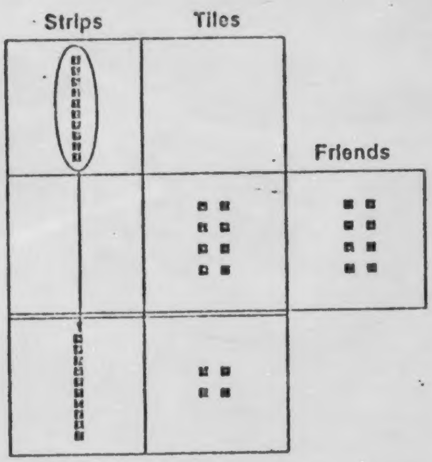
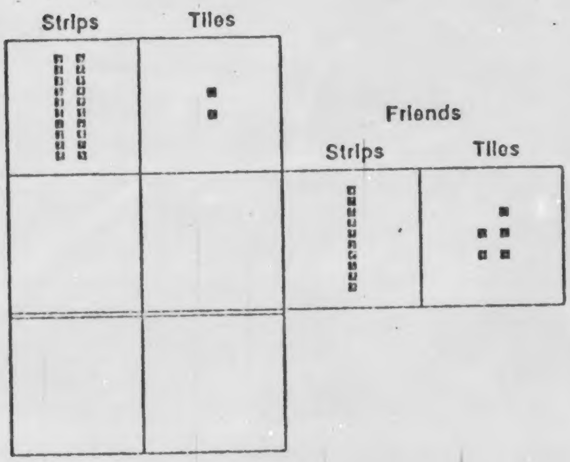
First, one tile for each friend is "taken away" from the original quantity. Next, the answer (first the ones place, then the tens) is brought down to its final position.


Strips
Tiles
Friends


Strips
Tiles
Friends

Example 2.

$$\begin{array}{r} 22 \\ -\underline{15} \end{array}$$


Strips
Tiles
Friends
Strips
Tiles

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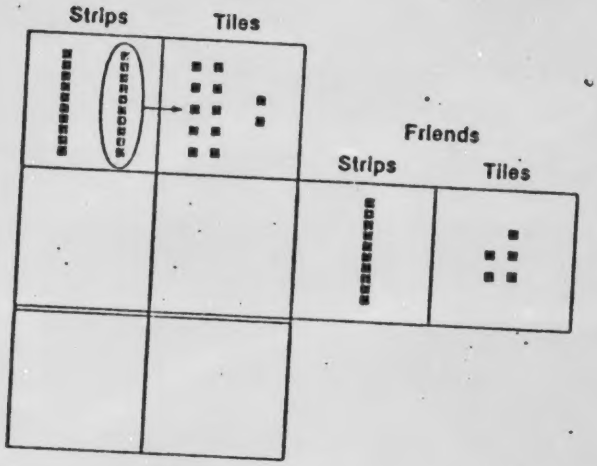
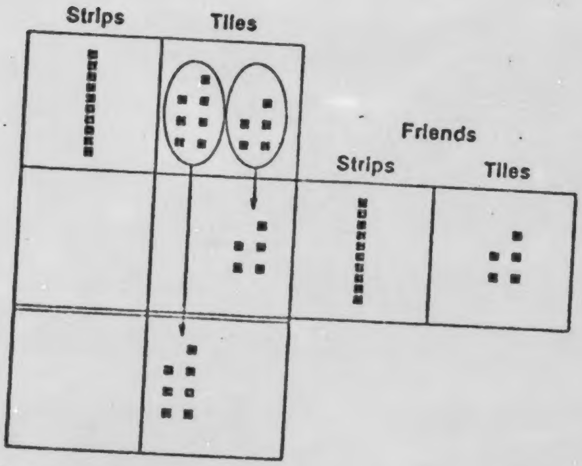
Step 1. Regroup tile strip to tiles place.


Strips
Tiles
Friends
Strips
Tiles

Step 2. Take away appropriate number of tiles (from the ones place) and bring the rest down to their final position.


Strips
Tiles
Friends
Strips
Tiles

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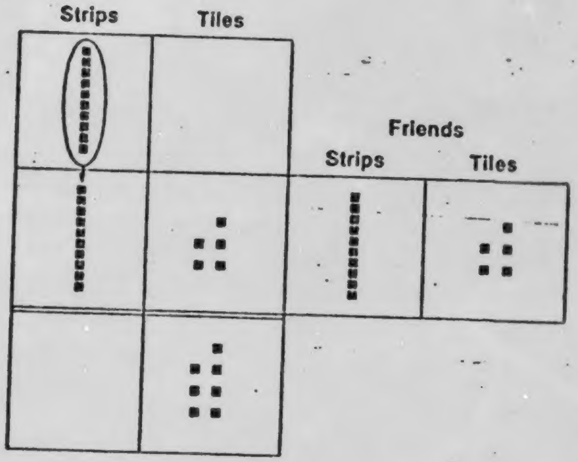
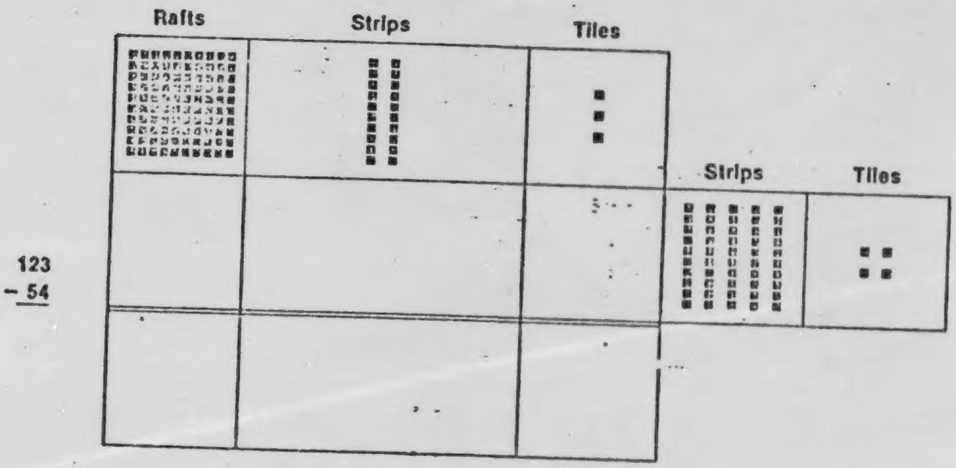
Step 3. Take away appropriate number of tile strips (from tens place) and bring any remaining strips down to the final-answer place (in this case, none).


Strips
Tiles
Friends
Strips
Tiles

Example 3.


Rafts
Strips
Tiles
Strips
Tiles
123
− 54

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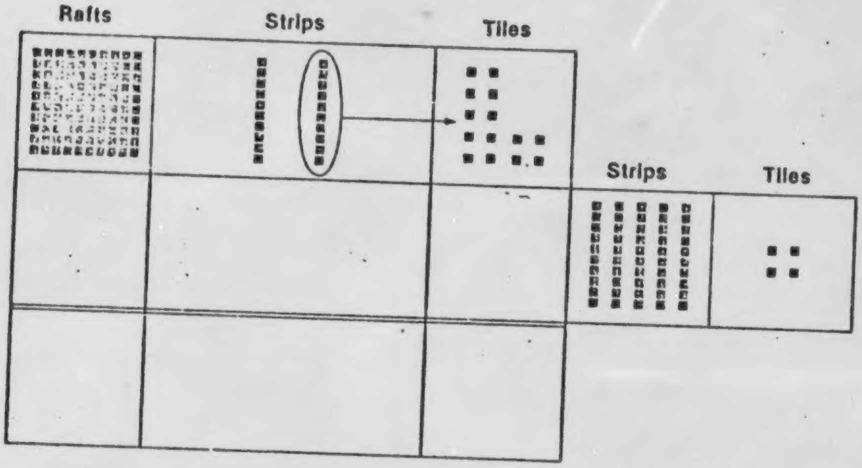
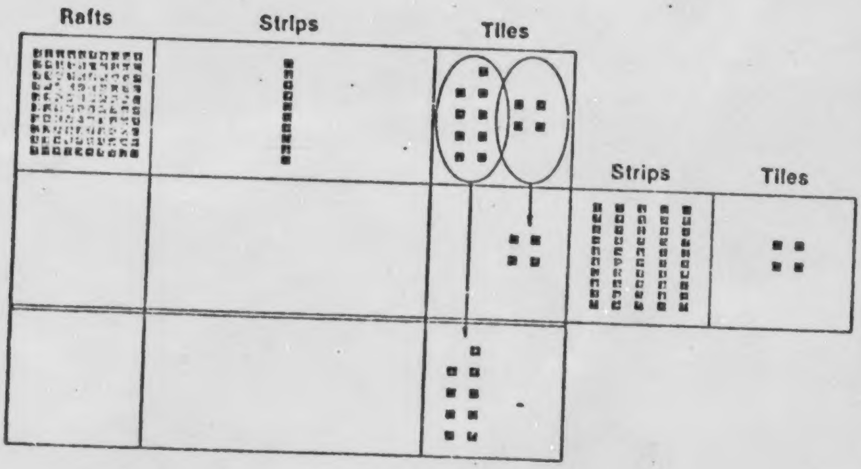
Step 1. Since there aren't enough loose tiles, regroup one tile strip and place tiles in ones column.


Rafts
Strips
Tiles
Strips
Tiles

Step 2. Subtract appropriate number of tiles and bring down remainder.


Rafts
Strips
Tiles
Strips
Tiles

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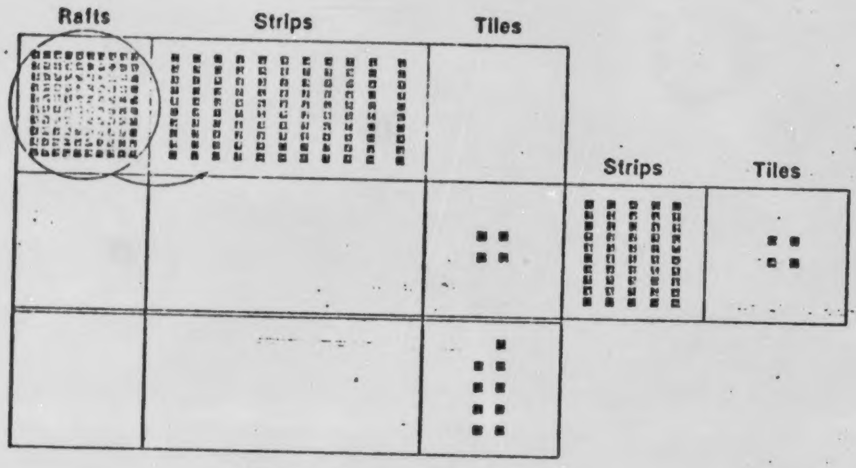
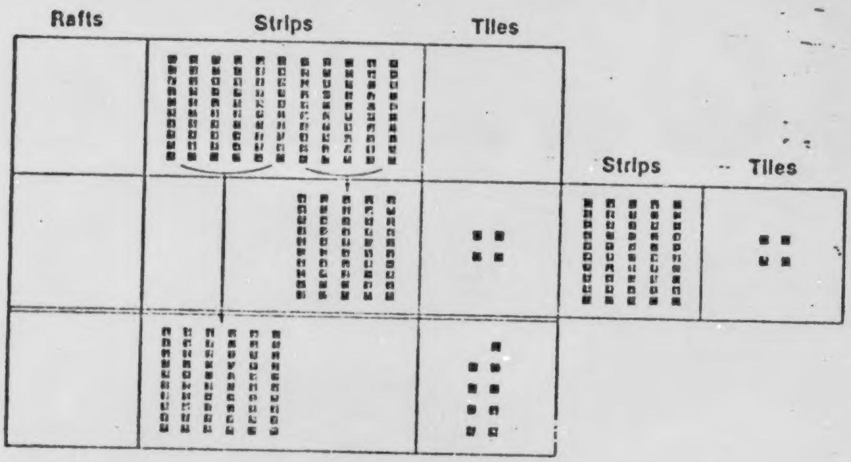
Step 3. Since there aren't enough tile strips, regroup one raft and place in tile-strip column.


Rafts
Strips
Tiles
Strips
Tiles

Step 4. Subtract appropriate number of tile strips and bring down remainder.


Rafts
Strips
Tiles
Strips
Tiles

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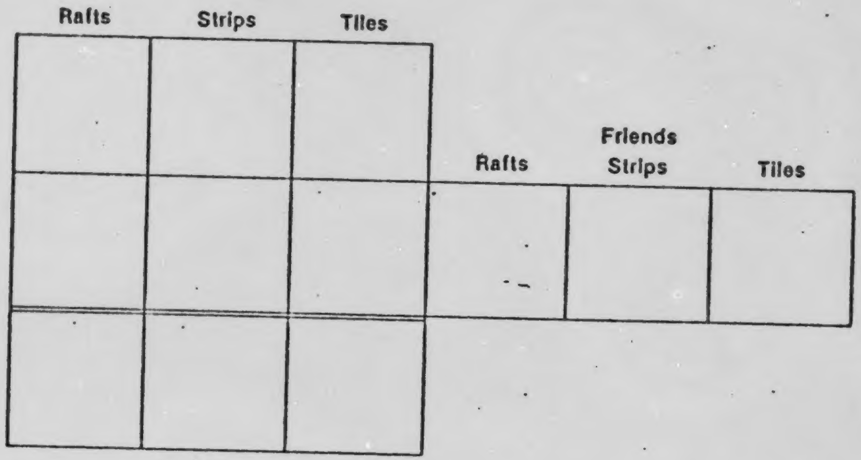
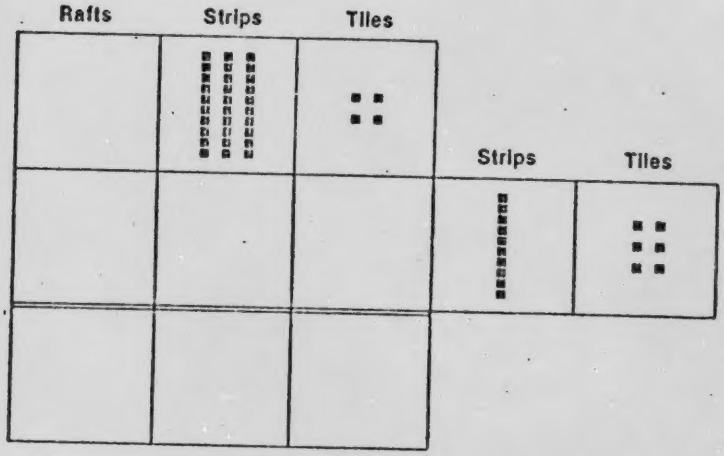
Activity 2-H2.
(concrete/ representational)

•*Materials* Tiles, tile strips, Record Sheet 2-H2.

•*Procedure* Repeat Activity 2-H1, but this time have students record their process using representational notation. Develop work sheets with the boards already drawn on them, i.e.,


Rafts
Strips
Tiles
Rafts
Friends
Strips
Tiles

For the problem 34 – 16, the concrete situation would look like this:


Rafts
Strips
Tiles
Strips
Tiles

and the representational record would look like this:

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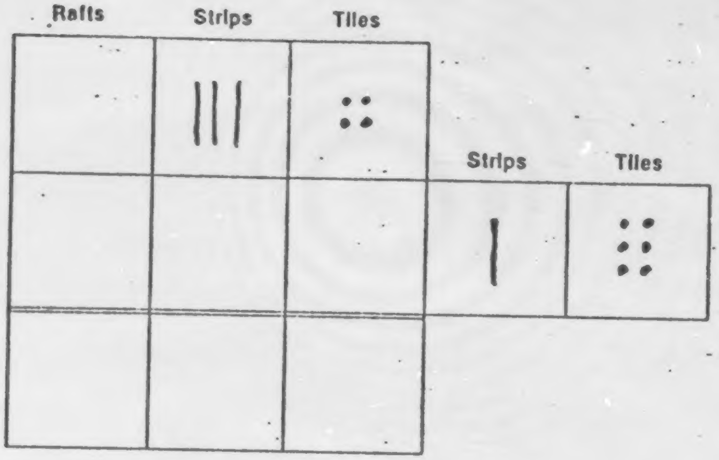
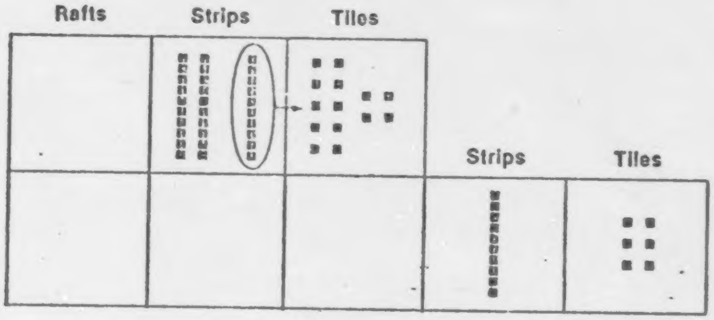
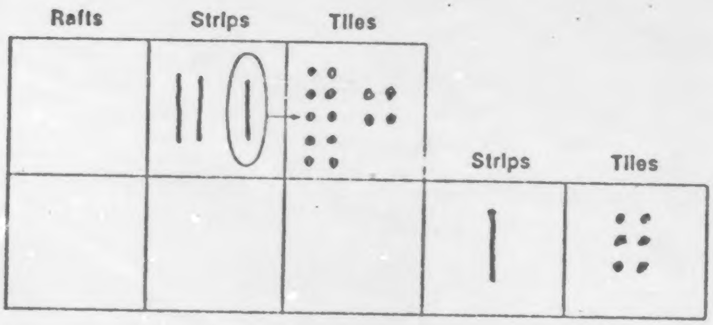

Rafts
Strips
Tiles
Strips
Tiles

First regroup tile strip to tiles place.


Rafts
Strips
Tiles
Strips
Tiles


Rafts
Strips
Tiles
Strips
Tiles

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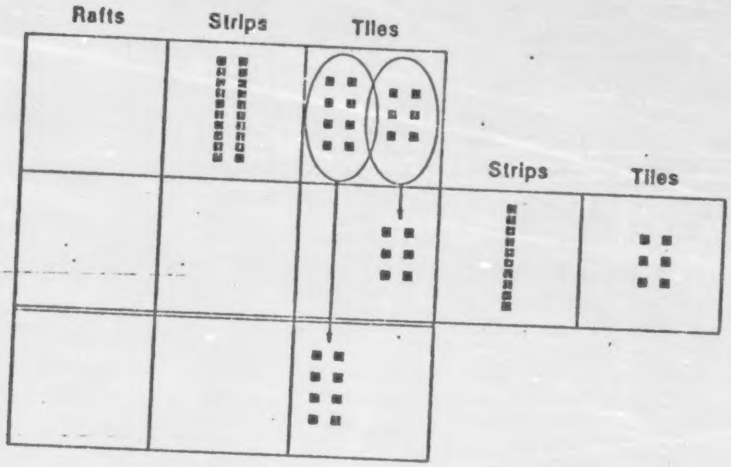
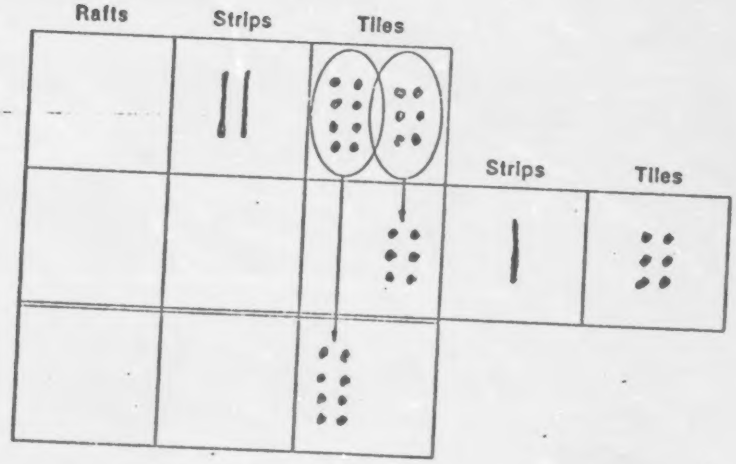
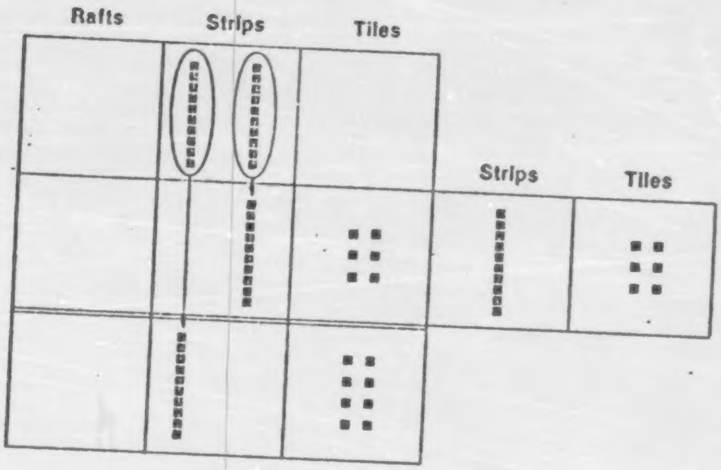
Next, subtract appropriate number of tiles and bring the answer down to the ones place.

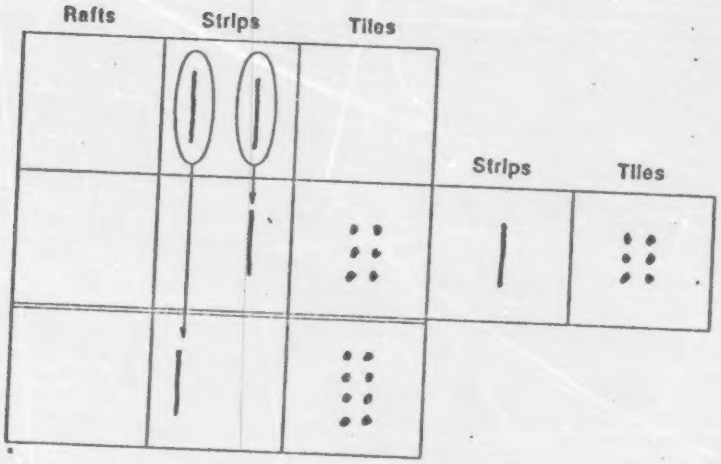

Rafts
Strips
Tiles
Strips
Tiles


Rafts
Strips
Tiles
Strips
Tiles

Souviney, R. et al. Mathmatters. Goodyear, 1978

Finally, subtract the appropriate number of tile strips and bring the remaining answer down to the tens place. The problem has now been solved.


Rafts
Strips
Tiles
Strips
Tiles


Rafts
Strips
Tiles
Strips
Tiles

Souviney, R. et al. Mathmatters. Goodyear, 1978.

SUBTRACTION

Name____

Rafts	Strips	Tiles

Friends

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Friends

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Friends

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Rafts	Strips	Tiles

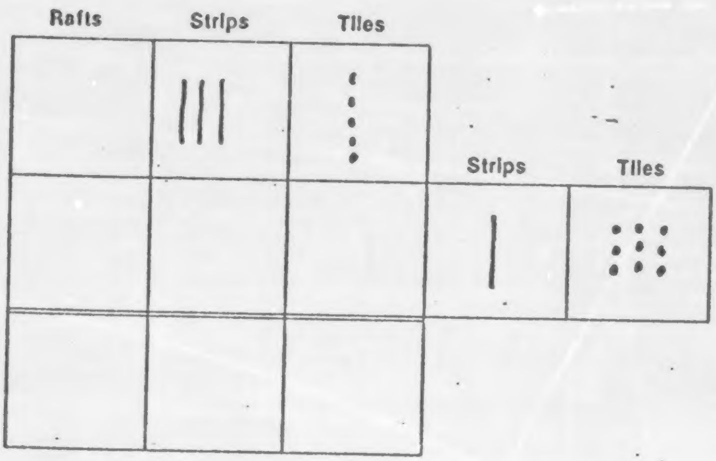
Souviney, R. et al. Mathmatters. Goodyear, 1978.

Activity 2-H3.
(representational/ symbolic)

•*Materials* Paper, tiles, tile strips, Record Sheet 2-H2.

•*Procedure* To familiarize students with symbolic notation for subtraction, have them use numerals to record their representational process on a separate sheet of paper.

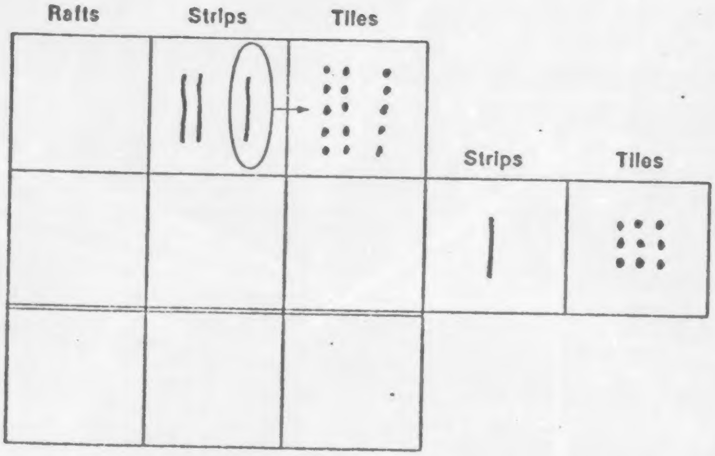
Original Problem:


Rafts
Strips
Tiles
Strips
Tiles

Record:

$$\begin{array}{rr} 3 & 5 \\ -1 & 9 \\ \hline \end{array}$$

Regroup:


Rafts
Strips
Tiles
Strips
Tiles

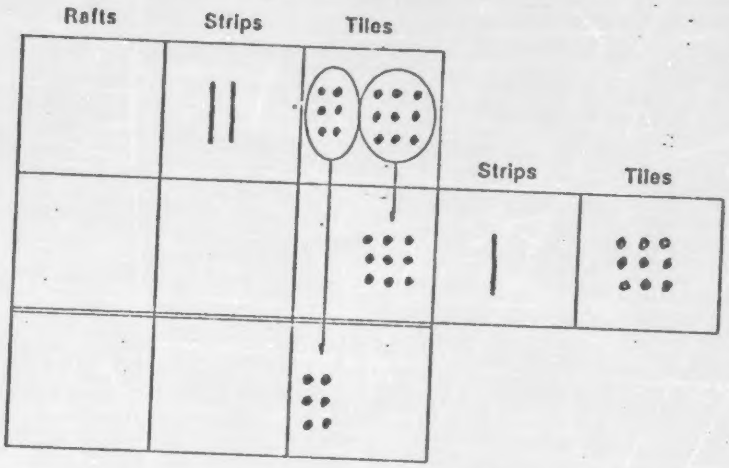
Record:

$$\begin{array}{rr} 2 & \\ \cancel{3} & 15 \\ -1 & 9 \\ \hline \end{array}$$

Souviney, R. et al. Mathmatters. Goodyear, 1978

Subtract and bring down answer (ones column).

Subtract:

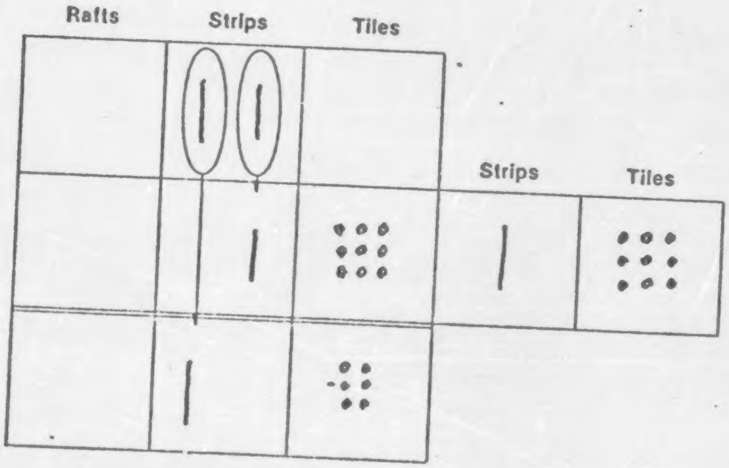

Rafts
Strips
Tiles
Strips
Tiles

Record:

$$\begin{array}{rr} & 2 \\ & \cancel{3}\ 15 \\ - & 1\ \ 9 \\ \hline & 6 \end{array}$$

Subtract and bring down answer (tens column).

Subtract:

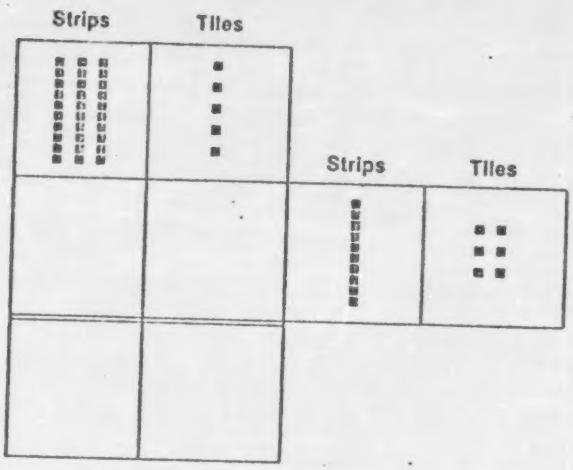

Rafts
Strips
Tiles
Strips
Tiles

Record:

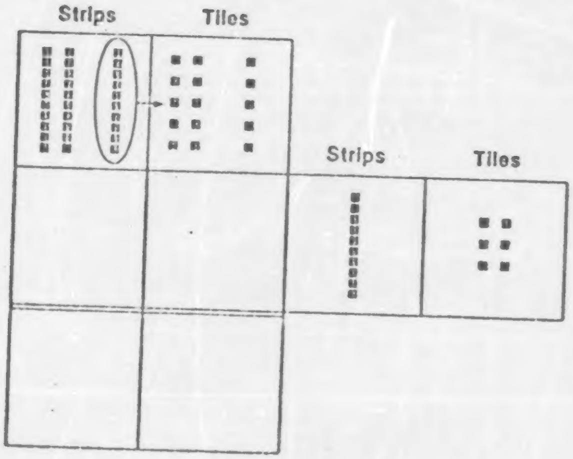
$$\begin{array}{rr} & 2 \\ & \cancel{3}\ 15 \\ - & 1\ \ 9 \\ \hline & 1\ \ 6 \end{array}$$

Souviney, R. et al. Mathmatters. Goodyear, 1978

After students have become familiar with the mechanics of symbolic notation, provide them with problems to be solved entirely at the symbolic level. Ask them to validate their answers by repeating the process with concrete materials or representational notation. For example, the problem 35 – 16 is given and the student finds an answer (19) at the symbolic level. He is then asked to validate his answer using tiles and tile strips. He sets up the problem,

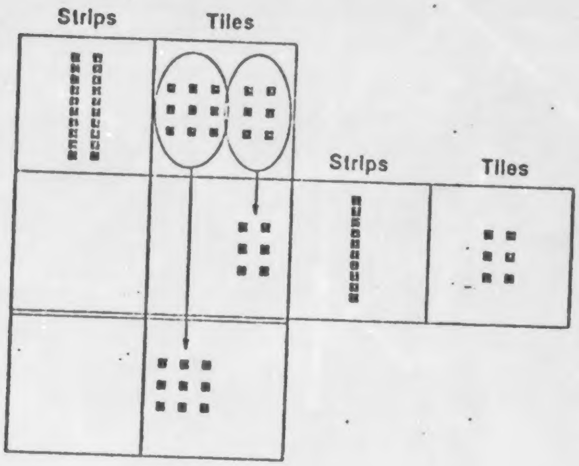


regroups,

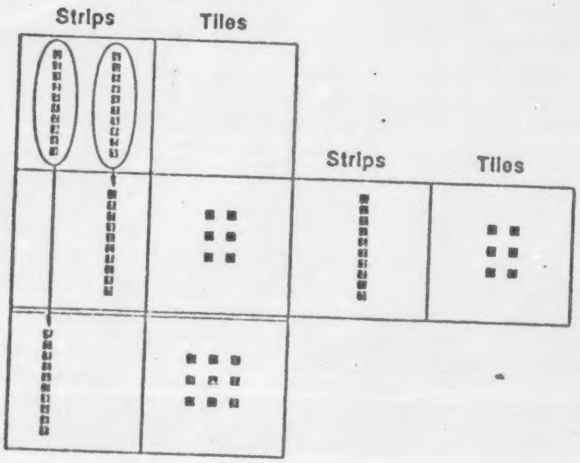


Souviney, R. et al. Mathmatters. Goodyear, 1978.

subtracts ones and brings down the answer.

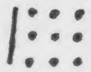

Strips
Tiles
Strips
Tiles

subtracts tens and brings down the answer.


Strips
Tiles
Strips
Tiles

Souviney, R. et al. Mathmatters. Goodyear, 1978.

The final answer is



i.e., 19--right!

Activity 2-H4.
(symbolic)

•*Materials* Pencils, Work Sheet 2-H4.

•*Procedure* Here is an interesting number property which can be used to spark students' interest in subtraction drills. Take any three-digit number (first and last digits must be different), reverse it, and subtract the smallest of the two from the largest. Next, reverse the difference and add. The answer will always be 1,089!

```
   452
 − 254
   198
 + 891
 1,089
```

Here's another example:

```
                                                    423
                                                  − 324 Reverse and Subtract
                                                    099
(In those cases you must include the zero.)      +  990 Reverse and Add
                                                  1,089 Eureka!!!
```

This exercise provides a self-checking mechanism for a student's subtraction. If his final sum adds up to 1,089, he knows that his subtraction was correct; if he gets something else, he will know something went wrong.

Souviney, R. et al. Mathmatters. Goodyear, 1978

Because the concept of place value is lost when the two digits are thought of as separate entities, children understandably encounter difficulty when they must eventually learn to regroup. Suddenly, the two digits must be related in some manner and cannot be dealt with separately.

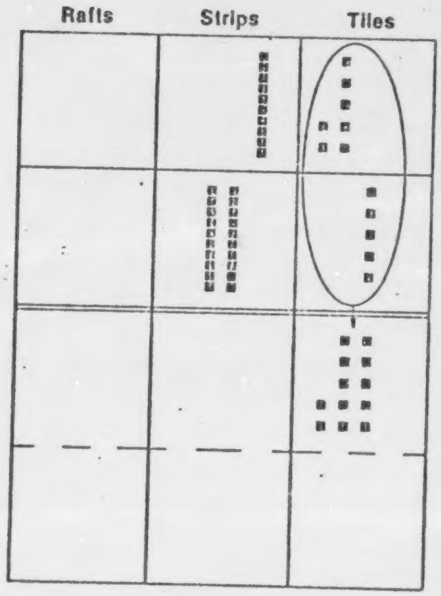
To avoid this confusion, two-digit addition with regrouping must be introduced simultaneously with nonregrouping situations. The concept should be presented in such a manner that both facets (regrouping and nonregrouping) are viewed as a natural and integral part of the process of addition.

Activity 2-G1.
(concrete)

•*Materials* Tiles, tile strips, Work Sheet 2-G1.

•*Procedure* Students are given boards and asked to work problems that are given to them verbally. Again, place values are represented on the board by columns for single tiles, strips of ten, and rafts of one hundred.

When asked, for example, to add seventeen and twenty-five, students first combine all the single tiles (ones) and move them to the answer block.



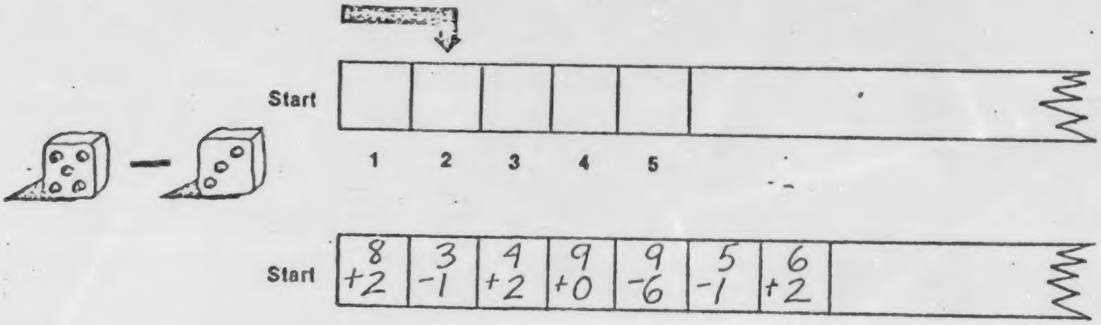
Students are then asked how many individual tiles there are in all and whether this is the "best way" (see Activity 2-B1) to show twelve. They should respond that the best way would be by means of a tile strip and two single tiles. At this, the teacher points out that students must therefore trade ten individual tiles for a tile strip and move the tile strip into the appropriate column.

Souviney, R. et al. Mathmatters. Goodyearyear, 1978.

Activity 2-F3.
(symbolic)

•*Materials* Dice games.

•*Procedure* Develop games similar to the addition drills. This time, the two dice are rolled and the larger number placed in front of the smaller. The difference indicates the number of moves or the next number to move to. Games combining addition and subtraction should also be prepared.



Aside from providing valuable practice, these games allow students to enjoy what might otherwise prove to be a dull period of reinforcement in the teaching of mathematics.

Concept G: Two-Place Addition with Regrouping

In most mathematics sequences, one generally finds two-place addition without regrouping introduced before and apart from problems that involve the regrouping process. However, these two types of problems involve only one process, namely, addition of multiple-place numbers. Children do not view adding twenty-two and thirteen pieces of candy as being different from adding twenty-two and nineteen pieces. Furthermore, when addition without regrouping is introduced separately, children fail to recognize the place-value relationship in two-digit numbers. The problem

$$\begin{array}{r}28\\+\ 41\\\hline\end{array}$$

becomes 8 + 1 and 2 + 4. (Ask a child which represents more in twenty-eight, the eight or the two.)

Souviney, R. et al. Mathmatters. Goodyear, 1978

I. (c) Solve multi-digit addition and subtraction problems using paper and pencil (using either an intermediate or standard algorithm).

An intermediate algorithm for multi digit addition is shown below:

$$\begin{array}{rl} 15 & \\ 18 & \\ \hline 13 & (5+8) \\ 20 & (10+10) \\ \hline 33 & \end{array}$$

An intermediate algorithm for multi digit subtraction is shown below:

$$\begin{array}{rcl} 33 & = & 20+13 \\ -18 & = & 10+\ 8 \\ \hline & & 10+\ 5 = 15 \end{array}$$

The standard algorithms are illustrated below:

$$\begin{array}{r} 1 \\ 15 \\ +18 \\ \hline 33 \end{array} \qquad \begin{array}{r} \overset{2}{\cancel{3}}\overset{13}{\cancel{3}} \\ -18 \\ \hline 15 \end{array}$$

NOTE: The 3 in the ones place is slashed out and 13 written above it. 2nd graders should show regrouping this way. They often do not "see" a 13 if a small one is inserted to the left of the 3. Also, when they have used the model first, this makes much more sense to them.

Either of these algorithms (or others the teacher may have developed) are appropriate for the child to demonstrate paper and pencil competencies.

CTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

II. (a) Students will prepare a graph from data they have collected and write at least one paragraph interpretting their results.

XAMPLES:

Have child measure water, birdseed, or and using cups, pints, quarts, etc. (see Iv. (c)) he child can cut out pictures of cups (many - identical) nd other containers. They would paste the various ontainers under the baseline of a graph in sequential rder and glue one cup above the baseline for each cup hat container will hold. They would then write a aragraph on their findings to display with the graph.

ee Open Math Resource Book pp 148-150, 227-228

orkjobs pp 222-223

Primary Math Lab pp 57-60

PERCEPTUAL AND REPRESENTATIONAL SPACE

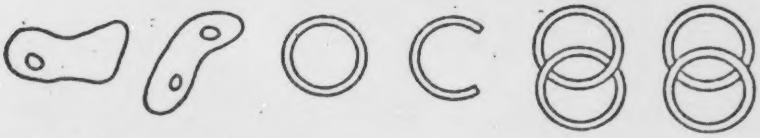
Purpose of Activity

Knowledge of space or the world around us develops in two ways—first, through what we perceive or what our senses tell us. Piaget refers to this concept as perceptual space. But our minds do not operate like cameras—photographing what our eyes see. Instead the mind must reconstruct or represent mentally such sensory data as obtained by touching or seeing. What the mind reconstructs is referred to as representational space, the second concept.

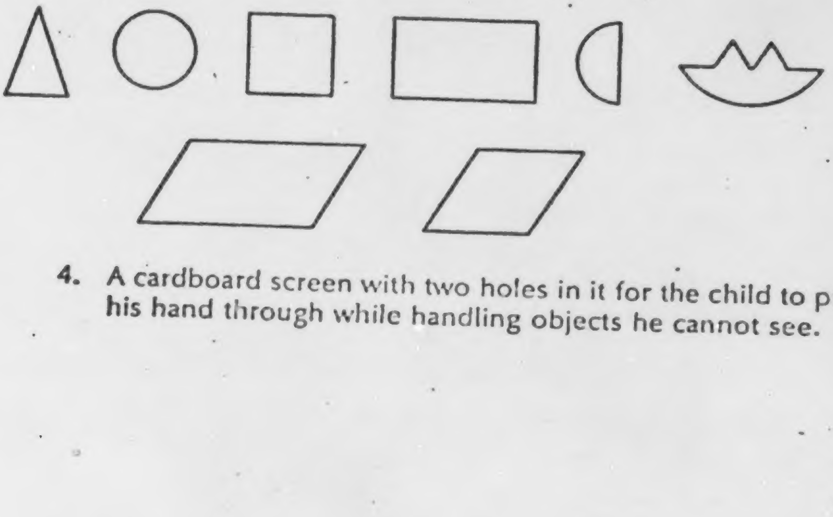
The purpose of this activity is to study children's ability to convert sensory or perceptual impressions of an object into mental representations of that object which, of course, is necessary for full understanding. What the mind "represents" may be and often is different from what is "seen" or "felt" by small children.

Materials Needed

1. A set of familiar objects such as a ball, comb, spoon, pencil.
2. A set of cardboard topological forms of a size convenient to handle.



3. A set of cardboard Euclidean shapes approximately 3 by 4 inches in size.



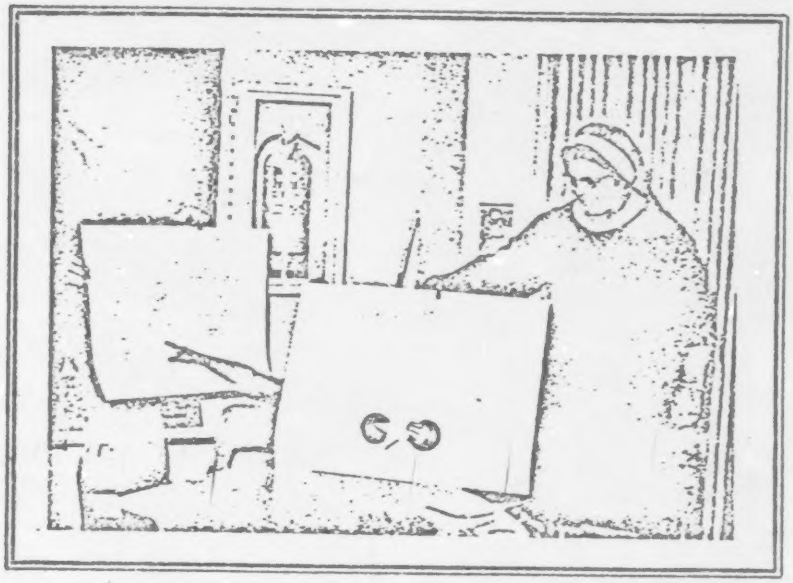
4. A cardboard screen with two holes in it for the child to put his hand through while handling objects he cannot see.

5. On a poster, pictures of objects the child is to handle (1–3).

Procedure

Ask the child to put his hands through the screen. Then, one at a time, give him objects to handle, asking him to point at the object on the poster.

For children 2 to 3 years old, use the familiar objects. For children 4 or older, begin with the topological figures. Piaget finds children able to make topological distinctions first—whether a figure is closed or open, has one or two holes, is intertwined or simply overlapping. It is more difficult to make Euclidean distinctions, such as noting the differences among triangles, squares, rectangles, parallelograms, and rhombi.



Levels of Performance

Stage 1. Familiar objects such as the comb are recognized, but children under 4 years of age are not coordinated in their action of handling the object. Coordination, of course, is necessary to construct a mental representation of the object.

Exploration is passive. No distinction is made between such Euclidean figures as the circle and square (since both are closed figures). Toward the end of Stage 1, distinctions are made between the topological figures such as whether the figure is open ⋐ or closed ◎ .

Stage 2. Usually, children from 4 to 6 years of age can distinguish between curved Euclidean shapes and those with straight sides, but no distinctions are made between such rectilinear figures as squares and rectangles. Neither are the curvilinear figures, such as circles and ellipses, differentiated from each other.

Explorations of the figures is more methodical, however; such shapes as the triangle and square begin to be differentiated by number of angles.

Stage 3. Beginning at approximately age 6, children are operational as far as basic Euclidean shapes and even more complex figures such as a cross are concerned. Account is now taken of order and distance between points on the figure.

Exploration is methodical. Furthermore, although it begins with perceptual activities as in the other stages, this stage now uses an operational method of *grouping* the elements perceived, such as corners or angles and the distances between them. This grouping is dependent on beginning at a fixed point on the figure, tracing the outline and being able to reverse the process, tracing back to the starting point, mentally grouping those sensory impressions into a coordinated whole at the representational level. This, children in the first stages cannot do.

Teaching Implications

Teachers need to know the stages through which children go in developing the ability to consider geometric ideas. Many teachers may also not be aware of the fact that understanding space is not just a perceptual activity. For example, teaching what a triangle is by the "show-and-tell" method does not at all ensure that the child can mentally represent the idea of a triangle and hence understand it.

Perceptual activities are the starting point but are not sufficient. Mental coordinating actions of these sensory impressions must take place. These actions must often be based on physical or motor action, such as tracing and retracing a figure with one's fingers. For some children the necessary mental coordination may not occur until 9 or 10 years of age.

Also, the task of picking a figure from a collection that the child has handled but not seen is less difficult than having to represent or draw the figure. However, children should also have activities in representing figures by drawing them when they are able. If motor coordination is poor, the child may be asked to make the figures out of matchsticks.

Copeland, R. W. Math Activities for Children. Merrill, 1979.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

VIII. (a) Identify 3 dimensional shapes of sphere, cylinder, cube, rectangula prism, triangular prism, and pyramid.

Do not rely on pictures of these objects. A bulletin board for each shape as it is presented ending with labeled examples of all shapes can be added to by the students.

Sphere -	use balls of various sizes and materials
Cylinders	paper tube or roller, cans, drinking glass, drum, straws, pill bottles,etc. PICTURES OF: Old fashion train engine, butane tanks, cakes, silos, garbage cans, above ground pool, etc.
Cubes (or square prism)	sugar, number blox, unifix cubes, boxes, etc.
Rectangular Prisms	boxes, books, file cabinets, bookcases, the classroom, etc.
Triangular Prisms	cheese wedge, door stopper wedge, sloped roof, (some wheelbarrows), (some cushions),etc.
Pyramid	tent, great pyramids

DRAWING (COPYING) GEOMETRICAL FIGURES

Purpose of Activity

This activity is to explore children's ability to copy basic geometrical figures.

Materials Needed

1. A card with figures as shown on the next page.[2]
2. A pencil.

Procedure

Ask the child to draw a man from memory both to put him at his ease and to get some idea of his drawing ability. Then ask him to make a copy of each of the figures on the card. These figures make distinctions between topological and Euclidean concepts of space.

Levels of Performance

Stage 0. At 2½ to 3 years of age, drawings are pure scribbles.

Stage 1A. At 3 to 4 years of age, the scribbles vary according to the model being copied. Open shapes such as Models 20 and 21 can be distinguished from closed shapes such as Models 4, 5, and 6.

Stage 1B. The drawings of these children, also 3 to 4 years of age, Piaget refers to as the first real drawings bearing some resemblance to the figures being copied. Definite distinctions are made between open and closed figures, but the closed figures are not distinguishable from each other. A circle, triangle, and square may all be drawn as an irregular closed figure.

Stage 2. Children aged 4 to 6 show progressive differentiation between the basic shapes, such as circles, squares, and triangles, in their drawings. Curved shapes are distinguished from straight-sided

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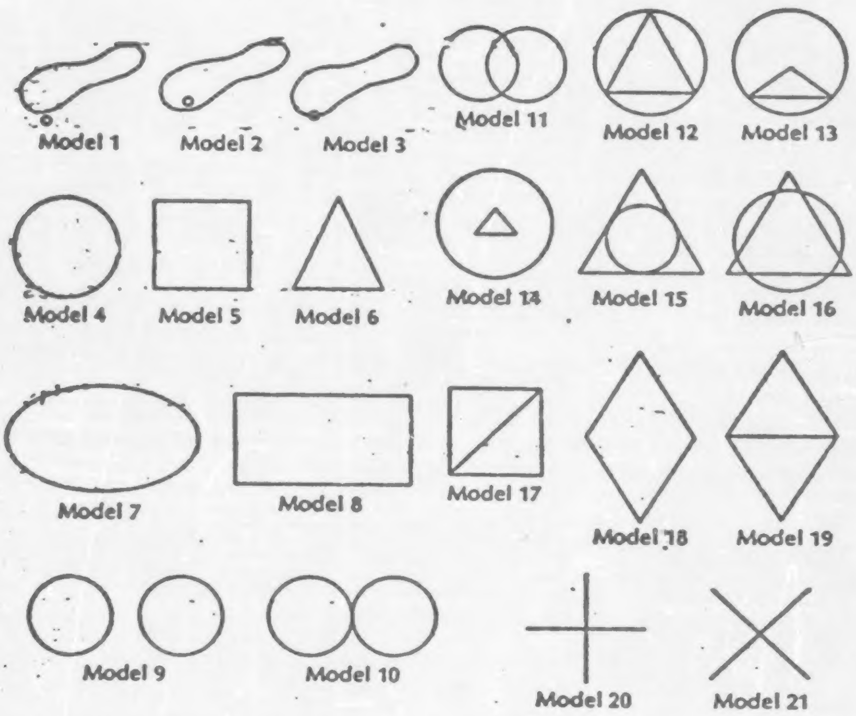
[2] Jean Piaget and Barbel Inhelder, *The Child's Conception of Space* (New York: W. W. Norton & Company, Inc., 1967), p. 54.


Model 1
Model 2
Model 3
Model 11
Model 12
Model 13
Model 4
Model 5
Model 6
Model 14
Model 15
Model 16
Model 7
Model 8
Model 17
Model 18
Model 19
Model 9
Model 10
Model 20
Model 21

ones, but little distinction is made at first between the curves or between the straight-sided figures.

A square and rectangle may appear to be the same. Gradually, differentiation is made in terms of number of angles. A triangle, for example, may be different from a square; a circle, different from an ellipse. The two crosses, Models 20 and 21, are drawn differently, indicating the recognition of difference between vertical, horizontal, and oblique. Circumscribed figures such as Models 13, 14, and 15 are drawn correctly as to shape, but the points of contact are not properly represented; that is, as touching, not touching or overlapping.

Stage 2B. The rhombus, Model 18, is now drawn correctly. Circumscribed figures such as Models 12, 13, and 15 (but not 16) are gradually mastered.

Stage 3. Children at this stage, 6 to 7 years of age, are able to draw all of the models correctly.

Teaching Implications

This activity provides a basis for determining in a clearly defined way what progress children are making in their ability to realize, as well as copy, basic spatial distinctions.

Children first make topological distinctions between figures. They distinguish "open" figures (Models 20 and 21) from "closed" figures, such as Models 4, 5, and 6. They are also able to make the topological distinctions in Models 1, 2, and 3: They can copy the small circle "outside," "inside," and "on the other figure." Some children, not functioning at the topological level, cannot even do this. In drawing a human figure, they may draw an eye outside the head or legs not attached to the body.

Later, children make Euclidean distinctions which show correctly the sides or angles; the length of sides; the size of angles; and vertical, horizontal, and oblique concepts. Thus they differentiate between such Euclidean shapes as squares, parallelograms, and trapezoids.

Most children will be ready at first-grade level to learn the basic shapes.

Copeland, R. W. Math Activities for Children. Merrill, 1979.

III. (b) Use pattern blocks and mirrors to create tessalating designs.

See Open Math for introductory pattern block activities.

Make a center for 3 or 4 students with a large supply of pattern blocks and 2 mirrors per child. Show them that they can hold the mirrors at angles by their designs to determine if they tessalate (repeat continuously).

Have children find examples of tessalating designs in their environment (eg. tiles, some wall paper, some carpets, some fabrics, bee hives, etc.)

Also use pattern blocks and designs to establish lines of symmetry.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

II. (c) Create and outline designs with tan grams for their peers.

Begin by introducing 3 or 4 easy shapes for children to find. (on the first one or two shapes include some interior lines.) (Use Marcy Cook's Tangram Mini Center to introduce.)

Challenge pupils to create their own designs. Have them outline the design and add it to the center for other children to solve.

Have two children work with a screen between them. Have one child describe a tangram design in fron of him to the second child who must reproduce it from the oral description.

EXTENSION: After solving a tangram design have children use the small triangle to mark off the entire design. How many small triangles does the design contain. (16 if all pieces used.)

X. (a) Explore number patterns involving both addition and subtraction.

See Mathmatters pages 254 - 275

Whats My Rule? (whole group game)

Leader (teacher or child) puts number function on the board with at least four numbers

eg.

?	
5	8
7	10
3	6
12	15
9	

and the results of applying a rule. Then a 5th # is placed on the right and the group is asked to raise their hand if they think they know what and how the rule would affect that number. (In the example given the child would say the next number is 12.) The first child called upon who gives the correct answer is the "ANSWER MAN OR ANSWER WOMAN." That person chooses a new number to go in the left hand column and is the one who says whether guessing classmates have a correct answer. (You may want to give the answer person 1 or 2 additional examples to solve to be sure rule is understood.) Have various answer persons explain rule in their own words to the class.

X. (b) Use colored rods to make stair cases of odd and even numbers.

Method One:

Demonstrate and ask children to make the following staircases: red, purple, dark green, brown, and orange; and white, light green, yellow, black, and blue. Ask them to talk about what they have made. How big are the steps? (Red on both staircases.) How could the steps be the same when different colors were used? If white were 1, what would the numbers on these staircases be? (2,4,6,8,and 10 and 1,3,5,7, and 9). What do we think of when we see those two groups of numbers? etc. (try to solicit idea of odd and even numbers from the group).

Method Two:

Give children the rods and the rule that 1 white = 1. Ask them to make a staircase of all even numbered rods and all odd numbered rods. Ask questions about what they see.

Method Three:

Ask children to make their own staircases with steps of red. When finished compare and see if they are the same (some will probably start with white and some with red - if so, go to questions in method one). If all students have the same staircase, challenge them to find another way to make a staircase with steps of red. Then ask series of questions such as those in method one.

FRACTIONS

Purpose of Activity

The purpose is to explore children's understanding of the quantitative relations involved in dividing a whole into equal parts.

Materials Needed

1. Paper cut into various shapes, such as circles, squares, and rectangles.
2. Six dolls.

Procedure

Show the child a circular piece of paper and two dolls and ask, "If this were a cake, could you cut it so that each doll would get the same amount? Can you show me with this pencil how the cake should be cut."

Vary the procedure with other shapes, such as a square and a rectangle.

Also use the same procedure with three dolls, then six dolls.

Levels of Performance

Stage 1. In children under 4 years of age, there is no plan and the divisions are not equal. The child is unable to relate the part to the whole once the cake is cut. The parts are now less or more than the original cake.

Stage 2. At Stage 2A, 4 to 6 years, children can divide small cakes into two equal parts, but not into three equal parts.

At Stages 2B and 3A, 6 to 7 years of age, children can divide into three equal parts, but it is a trial-and-error process; for example, drawing a line and then erasing it to redraw the line in a different position. These children do understand conservation of the whole—that once the cake has been cut it is still the same amount of cake.

Stage 3. At Stage 3B, around 10 years of age, children can with

Dividing a "cake" for three dolls.

some assurance tackle the problem of dividing the cake into six equal parts. They may, for example, first divide it into three equal parts and then divide each of the three into two equal parts.

Teaching Implications

Children are often introduced to fractions by being "shown" something divided and "told" the name of each part. Then they are given pictures divided into parts and asked to name the parts.

Children should do the dividing as well as naming the resulting fractional part. As this activity demonstrates, it is not as easy as one might think.

Copeland, R. W. Math Activities for Children. Merrill, 1979.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
(b) Handle and count fractional models (eg. egg cartons or fraction bars) to amounts larger than one whole (eg. five thirds).	
Have each child paint their own set of egg carton fracions which you have cut. Establish a color pattern and display it. All fractions of the same size must be the same color.	egg cartons)12 or 30 egg size) knife or cutting boa 4 colors - tempra paint
(Note: use only halfs, fourths, thirds and wholes. Make both models of ½. If using the 30 egg carton with 24 as a whole, make both models of ¼ also.)	
Hand a child a ½ (egg carton or fraction bar) and ask him/her what she/he has. Hand him/her a second ½ and repeat the question (answer 2/2). Hand him/her a third ½ and repeat the question (answer 3/2. Continue.	egg carton fractions
Bring out models for one whole and find how many wholes are the same size as the pupil's halfs. Establish the concept of different names or labels for the same amount of number. (Also introduces pupil to the idea that we use fractions to talk about amounts greater than one whole.)	
(Note: sixths, twelfths, and eighths if using 30 egg flats.)	change"4 colors to 6 or 7 colors".

Bilingual Third Grade Math Competencies for the Concrete-Operational Child

[illegible]. Classification (Logic/Comparing/Ordering.)

"The student will have experiences classifying, comparing, ordering; the use of number and reversibility allows the student to extend these activities through numerical descriptions."

(a) Classify objects by size, color, and the four basic geometric shapes (circles, squares, rectangles, triangles) and classify sets by the number of members.

(b) Compare sets of objects and verbalize about their size.

II. Numeration

"The student will have experiences with numeration; extensive use of numeration models will be used with many grouping and trading activities for developing the place value concept."

(a) Group bundles of over 100 counting sticks into 100's, 10's, and 1's.

(b) Participate in activities using manipulatives which are based on size but require trading rather than bundling.

(c) Participate in activities using manipulatives with arbitrary classification (not based on size, e.g., colored chips, soda pop bottle lids, etc.)

(d) Activities that involve representing numbers on an open topped abacus.

(e) Count forward by 2's, 5's and 10's from any given starting number.

(f) Count backwards from 100 by 10's, 5's, 2's, and 1's.

III. Notation

"The child will learn the written symbols associated with numerals, operations, and measurements after having concrete experiences with the concept and developing an oral label for the concept."

(a) The child will read and write the numerals from 0 to 9.

(b) The child will read and write the numerals from 10 to 100.

(c) The child will read and write the symbols for "greater than" "less than" and "equal to."

(d) The child will read and write the equations for the basic facts with sums to 18 in addition and subtraction (numerals, operation signs, equal sign.)

(e) The child will begin writing number sentences expressing inequalities with numbers of ninety-nine and less (numerals, operation signs, greater than and less than signs.)

(f) Pupil will write numerals from 0- 9,999 when shown a manipulative model of the number.

(g) Pupil will write numerals from 0- 9,999 from oral dictation.

(h) The child will read and write the equations for the basic multiplication and division facts (numerals, operation signs, equal sign.)

(i) Students will read and write basic division facts in algorithm form.

IV. Measurement

"The student will have experiences with measurement; early experiences with non-standard measurement will be followed by many activities at every level using both the metric and English systems for length, mass, capacity, area, and temperature. The measurement of time will include sequencing, and many activities of short (hours, minutes, seconds) and long (day, week, month, season, year) duration.

(a) The child will estimate length of real objects to nearest appropriate unit.

(b) The child will determine area of rectangles using one inch and one centimeter graph paper.

(c) Find the volume of various shaped boxes with inch and/or centimeter cubes.

(d) The child will estimate mass of real objects to nearest appropriate unit.

(e) Read time to nearest minute.

(f) Money: determine change to be made from purchases of specific amounts.

(g) Read a calendar.

V Basic Facts

"The student will have experiences with basic facts; once a child is able to demonstrate an understanding of an operation through the use of concrete materials, drill activities for memorizing the basic facts will be implemented."

(a) Review all addition and subtraction facts.

(b) Use real objects to illustrate multiplication with products through 100.

(c) Recall multiplication facts with factors of 10,5,2, or 9.

(d) Use real objects to illustrate division as the inverse of multiplication.

VI. Algorithms for Operations

"The student will have experiences with algorithms for operations; introduction for multidigit numerals via concrete materials and to be taught as a way of recording the concrete experience."

(a) Use real objects to add and subtract multi-digit numbers.

(b) Record multi-digit addition and subtraction problems from manipulative model.

(c) Solve multi-digit addition and subtraction problems using paper and pencil (using either an intermediate or standard algorithm.)

VII. Graphing

"The student will have experiences organizing and presenting data; quantities will be graphically represented in numerical form as well as pictorial."

Students will prepare a bar graph from data they have collected and write at least one paragraph interpreting their results.

VIII. Geometry

"The student will have experiences with shapes and space - extensive work with geoboards in exploring measurement of perimeter, area and angles."

(a) Construct polyhedra from various materials.

(b) Use pattern blocks and mirrors to create a tessalating design.

(c) Create and outline designs with tangrams for their peers.

IX. Patterns

"The student will have experiences with patterns; patterning will be continued with concrete materials (e.g., attribute blocks) but will now include numerical patterns as well."

(a) Explore number patterns involving both addition and subtraction.

(b) Use colored rods to make stair cases of odd and even numbers.

(c) Explore non-linear patterns via pattern blocks.

X. Fractions

"The student will have experiences with fractions; early concrete experiences will develop the language of fractions (including physically additing fractional parts with the same name and naming the new quantity.) Numeration will be introduced once the child can divide."

(a) Identify one-half, one-third, one-fourth, one-sixth, and one-eighth of such things as a cup of water and of the basic geometric shapes.

(b) Handle and count fractional models (e.g., egg cartons or fraction bars) to amounts larger than one whole (e.g., five-thirds.)

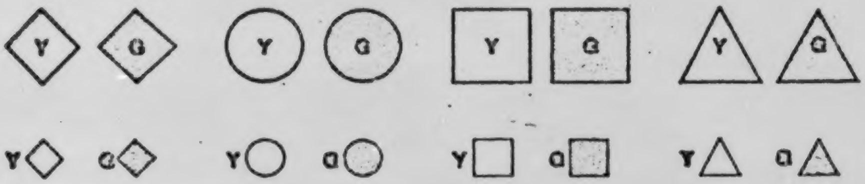
(c) Introduce notation for fractional numbers.

Logical Classification

(Number Readiness)

•*Purpose:* This activity evaluates a student's ability to sort (classify) objects consistently based on observable likenesses and differences.

•*Materials:* A set of shapes similar to those below. Each shape should be provided in two colors.



•***Procedure:*** Have the student dump the blocks onto the table and ask him to sort them into two groups so that each group is alike in some way. Children who do not classify logically will have difficulty with this problem. They may change their grouping criteria in the middle of the problem or may simply make pretty patterns with the shapes. In most cases, children between the ages of two and six fall into this category. Around age six, children generally become more adept at solving this problem but nevertheless require some help with organization in order to ensure that all the blocks will fit into one group or the other. Between the ages of six and eight, most children are able to solve this problem independently and without hesitation, providing complete descriptions of their grouping criteria.

Until a child can classify logically, it is not advisable to introduce number concepts at the abstract level. Number concepts should remain implicit in the concrete materials and not formally defined by means of symbols.

Souviney, R. et al. Mathmatters. Goodyear, 1978.

ACTIVITIES FOR LEARNING THE COMPETENCY — MATERIALS

I. (a) Classify objects by size, color, and the four basic geometric shapes and classify sets by numbers of members.

Also See: 2nd grade activities.

Attribute trains: — attribute pieces

(Example)

Lay down a large red square in the center of the table. On one side place a small red square. On other side place a large blue square. Above the center place a large red triangle and below it a large red circle. Ask students to tell how each of these pieces is like the large red square and how they are different. Note that they are all alike in two ways and different in one way. Point to the small red square. Ask what other pieces have two attribute in common with that piece and one attribute different (eg. small red circle or triangle, small square of another color, and large red square). Ask them to choose one new piece that meets the criteria and place it on the opposite side of the small red square from the large red square. Point to the large red triangle and ask what you could place above it that would continue the train (large red rectangle, small red triangle, large triangle of another color). Continue with the large red circle and the large blue square. Have students take turns adding pieces to the train, each time telling what two attributes are in common and which attribute is different.

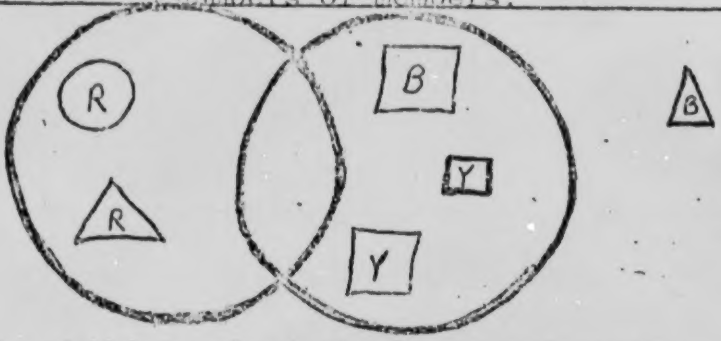
Have children repeat this activity with a new starting piece.

Vienes Diagrams — attribute pieces, yarn loops

Lay two loops of yarn in overlapping circles. Place several attribute blocks as shown below:

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

I. (a) cont. Classify objects by size, color, and the four basic geometric shapes and classify sets by numbers of members.



Ask the child to place the large red square in its correct spot.

What's My Rule?

Attribute piece
Yarn loops

Make up cards with some of the following rules:

cards (with rules)

All Red
All Circles
All Squares
All Triangles
All Blue
All Yellow
All Small
All Large

Make two overlapping circles with the yarn. Place one of the rule cards face down in each circle (but not in the intersection). Divide the group into two teams. Place one attribute piece in each circle and two pieces outside of the yarn. Place all other pieces on chalk tray of chalkboard. Have teams take turn (each member must play in order) selecting an attribute piece and placing it on the table where they believe it belongs. If they are incorrect the piece is returned to the chalk tray. If they are correct, the team may huddle & make one guess at what is on one of the rule cards. Teams earn one point for each correctly placed attribute piece and five points for guessing a rule.

VARIATION: After children are very competent extend the activity by using 3 loops and making cards for intersections, (eg. all red squares).

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
I. (b) Compare sets of objects and verbalize about their size.	
Workjobs 144-145	
Give children a set of attribute blocks and tell them to sort the blocks into two groups (give them two boxes) so thatevery thing in each box is related. After they have finished ask "How are these alike?" "Can you sort them into two boxes again but in a different way?" (Continue this until they have sorted by all double attributes such as large and small, thick and thin, round and not round.) Next give them the number of boxes that are equal to the number of colors and shapes in the set and ask them to continue grouping. Always ask how the groups are chosen, "How are these alike?" (If attribute blocks are not available assemble a collection of cars, pencils, and blocks - some red, some blue, some yellow for instance, and some larger than others. Also, attribute pieces may be made from colored construction paper and laminated, however, they will lack the attribute of thick and thin.)	Attribute pieces
Bring out a variety of objects. Begin to put the objects into two sets, perhaps on a piece of tagboard marked into twotwo sections. Into one set put things with a common property or identity - animals, perhaps - and into the other set objects that are not animals. As soon as they think they know it, have the children tell the name of either set. They will easily name the set of animals; the difficulty comes in finding a name for the other set. They may suggest names like "things" or "toys" or "objects", but if any of these were the name of the set then all the animals would be members of that set, too, and they are not. If no satisfactory name can be suggested, put that display aside, still in view, and begin another partitioning, such as "yellow" and "not yellow". Again seek names for the two sets. If several such examples do not elicit the desired response from the children, return to the set of animals and say, "Listen while I think how I'll decide where this ones goes. Is it an animal? Yes; so I'll put it here with the set of animals. Is this an animal? No; so I'll put it with that set. This is the set of animals. What could be a name for that set?"	Numerous small objects

Competency/coordinated activities - 3rd grade
Concrete-operational

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
I. (b) Compare sets of objects and verbalize about their size.	
Display sets of objects whose only common attribute is the number of members (eg. 5 gold stars, 5 red balls, 5 blue rectangular regions, 5 pink erasers, 5 yellow pencils, etc.) Ask the children to tell you what all of the sets have in common. (If all other attributes have been controlled, you should have no trouble eliciting the attribute of number form a group of children, however one child alone may not be able to recognize number as an attribute.)	Sets of small objects (eg. gol stars, red balls blue rectangular regions, yellow pencils, silver jacks, etc.)
Relate to data gathering activities. For example, have children go to parking lot to determine if there are more red cars or more blue cars or to compare the license plates.	
Use large numbers by comparing quart jars of lima beans and unpopped pop corn. Which has the largest number? Why? Is there the same amount? In what sense? (Lead to the idea of various ways to quantify - especially by counting and measuring.)	

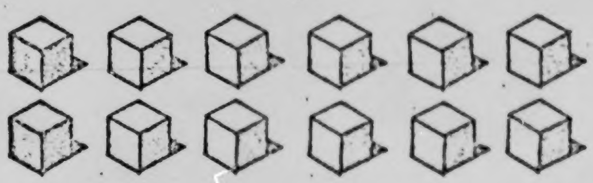
Conservation of Number

(*Number Readiness*)

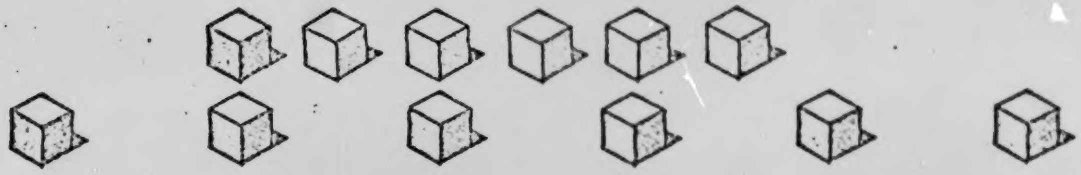
•*Purpose:* This activity evaluates a student's understanding of the invariance (conservation) of the number of objects in a set when the elements are moved about.

•*Materials:* Twelve identical wooden cubes.

•*Procedure:* Make a row of six cubes in front of you. Give the rest of the cubes to the child and ask him if he has the same number of cubes as you have. Have him prove it by putting the blocks in one-to-one correspondence, as below.



Now spread his blocks out and ask him whether you both still have the same amount.



Young children (four to five years of age) have difficulty determining whether the numbers of blocks are equal and generally cannot place the two sets in one-to-one correspondence in the first place. Even if they can initially determine equivilence by putting the two sets into one-to-one correspondence, when the rows are transformed their decisions are ruled by perceptual considerations; therefore, they believe that the spread-out row contains more cubes. Over the next two years, they will become capable of putting the blocks in one-to-one correspondence, but the decisions will still be ruled by perceptual appearances. By the age of seven, children are usually able to solve this problem correctly. We then say that the child conserves number.

As with classification and seriation, it is not recommended that children be introduced to number concepts at the abstract level until they conserve number. Once children are dealing effectively with classification, seriation, and conservation of number, they are ready to be introduced to the abstract notion of number and the associated symbols.

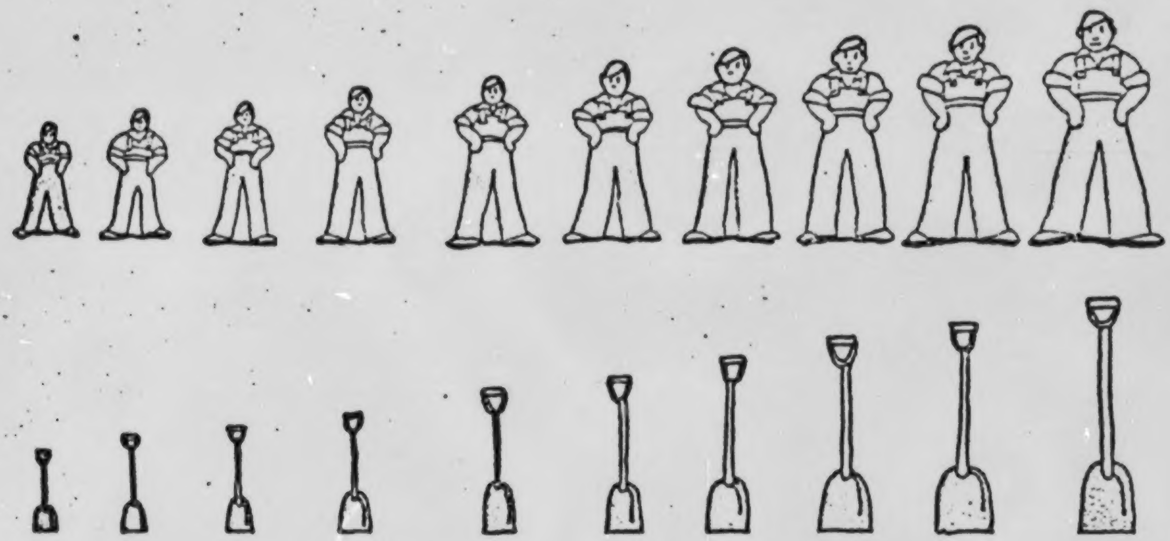
Souviney, R. et al. Mathmatters. Goodyear, 1978.

Seriation

(Number Readiness)

•*Purpose:* This activity evaluates a student's ability to place objects in serial order based on some attribute of size.

•*Materials:* Ten paper cutouts of workers gradually increasing in size, with the largest twice as big as the smallest. In addition, you will need ten paper cutouts of shovels gradually increasing in size, with the largest three times as big as the smallest.



•***Procedure:*** Ask the child to arrange the workers in order from small to large. Then have him position the shovels so that each worker gets the right-size shovel. Mix the workers up and ask the child to rearrange the shovels accordingly. Next give him the shovels, point to one of the workers, and ask the child to produce the appropriate shovel.

Young children (two to five years old) are unable to seriate either the workers or the shovels. Over the next two years, most children develop the ability to solve the three seriation problems, primarily by trial-and-error methods. Generally abilities necessary to solve the three problems are acquired in the order in which the problems have been listed above. After age six, most children readily solve all three seriation problems without using trial and error.

Until a child can deal effectively with concrete seriation problems, number concepts should not be introduced at the abstract (symbolic) level. Seriation is very closely tied to the concepts of cardinal and ordinal numbers and should therefore be prerequisite to the formal introduction of these concepts.

Souviney, R. et al. Mathmatters. Goodyear, 1978.

Competency/coordinated activities - 3rd grade
Concrete-operational

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

II. (a) Group bundles of over 100 counting sticks into 100's, 10's, and 1's.

Banker's Bundling Game (from A.S.U. lab manual) 4-6 players

1) Players each role a dice. Player with highest # is designated as banker and takes charge of the counting sticks.

2) Player to the lect of the banker begins. Player throws a dice, asks banker for that # of ONES, and bundles when possible. (Trading banker 10 ones for a bundle of 10 speeds up game but provides less practice in bundling sticks.)

3) Play continues in this manner, each player except the banker in turn throwing a dice, receiving ONES, and bundling where possible.

4) The winner is the first person who makes a HUNDREDS bundle. Winner becomes banker if played again.

5) VARIATION: Each player begins with a HUNDRED bundles and discards the number of sticks thrown on dice, unbundling when necessary. Winner is first player rid of all sticks.

counting stick
(in bundles o
1, 10 and 100

dice
rubber bands

Teacher might use sticks on overhead projector and have children repeat at desk - goal: forcing children to use mental imagery of the sticks.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

II. (b) Participate in activities using manipulatives which are based on size but require trading rather than bundling

Materials which meet these criteria include Base-Ten Blocks, number blocks; Diene's Multibase blocks; Cuisenaire Rods, Squares, and cubes; Bean Sticks; and tile strips. The latter two are "home made" manipulatives as follows.

Bean sticks are student constructed materials created by gluing exactly 10 dried pinto beans on a tongue depressor. Ten sticks are glued together into a raft of 100 beans. Single beans are used for ones.

Tile Strips are student constructed materials much like bean sticks. Small craft-type ceramic tiles (or large 1" tiles if necessary) are glued onto strips, 10x10 arrays are glued onto squares using rows of alternating colors for easy counting by tens. (Large tile outlets generally have cases of off-color tile or tiles which have come loose from their cloth backing which they will sell cheaply.)

ACTIVITIES:

Bankers game (See II(a)) can be played with any of these materials. The game can be speeded up (and/or played to 1000 using the commercial materials) by having dice of two colors - one for the number of ones and one for the number of tens. (Variation - let children throw dice and choose which will represent ones and which will represent tens.)

Base 10 blocks
Number blocks
Multibase blocks
Bean Sticks or
Tile Strips

See Also:

Mathmatters pp 54-117

Open Math Chapter 2 and 3

Workjobs I pp 146-147

Dice
(or sponge cubes
with numerals writte
on each face)

IMPORTANT: Activities in II.(a) are pre=requisite to use of these materials.

ACTIVITIES FOR LEARNING THE COMPETENCY | MATERIALS

II. (b) cont. Participate in activities using manipulatives which are based on size but require trading rather than bundling.

If students have actually manipulated one of the commercial models (eg. Multibase blocks) then many additional activities may be used to induce the representational level in the children. Some examples are:

a) Teacher demonstrates with real objects on overhead and directs attention to the screen (best if specific number is represented and children are then required to represent it concretely at their desk).

b) Pictures of blocks may be shown on screen via overhead or on activity cards. (Simple line drawings black and white masters are available from Creative Publications - may be more appropriate than the photographs usually shown in textbooks.)

c) Allow child to use the DLM Base ten Rubber Stamps to represent numbers and verbalize about what they have represented. (Allow students at a center to stamp whatever they choose on a page but then they must write the numeral they have represented.

ACTIVITIES FOR LEARNING THE COMPETENCY **MATERIALS**

II. (c) Participate in activities using manipulatives with arbitrary classification (not based on size).

MATERIALS

Materials that meet these criteria include commercial Chip Trading Materials, poker chips, a large assortment of lids of particular brands of soda (where coke lids are I, Pepsi lids are 10, etc. Dr. Pepper lids are 100, etc. based on quantity of each brand), construction paper squares of various colors, different types of dried beans (where pinto beans are 1, navy beans 10, lima beans are 100, etc) or any other system that students and teacher can come up with.

Colored chips
pop lids
construction pape
beans
etc.

ACTIVITIES

Activities can be those used in II (a) and (b) but with the new materials.

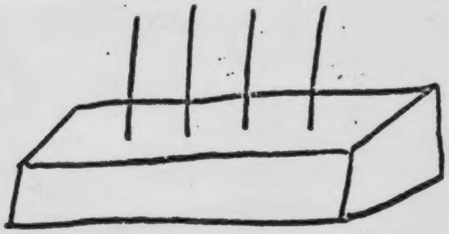
NOTE: The activities with materials described in II (a) and II (b) are pre-requisite to these materials.

II. (d) Activities that involve representing numbers on an open topped abacus.

The Abacus:

Have students make their own abacus by spacing 4 nails evenly with enough room between for washers or beads on the nails.

scrap wood
concrete or
finishing nails
washers or
beads



Have the children place 9 beads or washers on each nail and mark the spot on the nail where they end with tape or paint. This makes it easy to see when there are over nine objects on a nail.

Explain to the children that we begin counting on an abacus by using the nail furthest to the right. Have students place beads or washers in unison with you, counting orally. When they reach ten tell them "This abacus has a special rule. Only nine washers (beads) can end up on one nail. Everytime you get more than nine - when you get ten on a nail - you have to trade. You trade 10 washers from the right hand nail to one washer on the next nail." Have everyone make the trade. Every child should now have one washer on the "tens" nail and none on the "ones" nail. Have children add one washer to the "ones" nail. "We now have one ten washer and one one washer. What number would we call this? Continue

II. (d) cont. Activities htat involve representing numbers on an open topped abacus.

eliciting the names of numbers until every child in the group has the pattern (usually by 30).

After children have the pattern for representing numbers through 99, begin the following: "Can everyone show 96 on their abacus?" Check that they are able to do so. If they have trouble ask how many tens there are in 96, then how many ones. "Everyone add one washer (bead) to the one's nail." "Now what number do you have?" Continue adding until they reach 99. "What will happen if you add one more?" "What do you have to do when you have 10 washers (beads) on that nail?" Lead children through trading 10 ones washer for one tens washer, if necessary. "Now how many washers (beads) do you have on the tens nail?" "Can you have ten washers on one nail?" "What do you think we should do about it?" After leading (if necessary) children to suggest trading in the ten washers for one on the next nail, remind them of what numbers they were working with (99 and 1 more). "What number do we have represented now?" The third nail should now be named the hundreds nail. Have students add one washer and name the amount as one hundred one (remember - no ands). Continue through one hundred twelve.

After children have mastered the above activities they should use the abacus to play the banking game (IIa,b,& c). After playing the banking game the children should use the abacus for multi-digit addition and subtraction with regrouping.

ACTIVITIES FOR LEARNING THE COMPETENCY | MATERIALS

II. (e) Count forward by 2's, 5's, adn 10's from any given starting number.

Concrete group activities:

Tens

Begin with two longs and 6 units. Ask the group to name the amount. Write it on the board. Have someone add a long to the set. Have students name the new amount. Record it on the board under the 26. Continue the activity through at least 86. Ask children to look at the board. Do they see a pattern? What have they been doing? Has this been a kind of counting? What are they counting by? Lead them to idea that they are counting by tens even through they did not begin with zero. The pattern is that while the tens increase by one the ones remain the same.

Numeration blocks or tile strips or bean sticks or cuisenaire rods

After this activity has been done a second time, ask the children to count by 10's from a number you write on the board. Write 37 on the board. "If I count by tens, what number will come next?" As they say each number, record it on the board. (Go past 100.)

Fives

Begin with one orange rod and one red rod (12). Ask children to name the amount. Write it on the board. Have someone add a yellow rod. Have students name the new amount. Record it on the board under the 12. Have them add a second yellow rod. Ask them to trade the two yellow rods for one orange rod. Now ask them to name the amount shown. Record the amount on the board. Continue through at least 42. Ask the children to look at the board. Do they see a pattern? What are they counting by? What happens to the ones when you count by fives? (They may need to begin with 3 or 4 ones and repeat the activity before they are able to answer this question.)

Cuisenaire rods

II. (e) cont. Count forward by 2's, 5's, and 10's from any given starting number.

After they have identified the pattern of counting by fives, write 39 on the board and ask children to tell you the succeeding numbers by 5's.

Twos

Begin with a representation for the number 12 in the model chosen. Ask children to name the amount. Record it on the board. Add one red rod (or two units of other models) and ask for the amount. Record it on the board. Continue the activity, regrouping where necessary, through at least 34. Ask the children to look at the board. Do they see a pattern? What are they counting by? What happens to the ones?

Cuisenaire rods or
Number blox or
Bean sticks or
Tile strips

Repeat the activity beginning with 17. (Go through 39.) Repeat your questions. What is different and what is alike? Lead them to realize that the pattern depends on whether you begin with an odd or even number.

Record on the board several numbers to begin counting by twos and have the class tell you what to write.

Board Races:

See V.(c) - Have several students at the board. Tell them the rule (eg. "Count by 5's"). Then give them a starting number (eg. 152). Allow students to record their numbers for a limited time (eg. 90 seconds) and call stop. Winner is the person who has accurately counted the furthest.

chalkboard
& chalk

Variation: Have two teams.. Use a relay system. Write a beginning number in two places and give a rule. Have students relay to board to write next counting number according to the rule.

Variation: Have several children at the board but give each a different beginning number.

ACTIVITIES FOR LEARNING THE COMPETENCY | MATERIALS

II. (f) Count backwards from 100 by 10's, 5's, 2's, and 1's.

Songs:

Use the tune 99 bottles of beer on the wall and change the words if desired (eg. 99 noisy kids on the bus, 99 noisy kids; one of those kids put off of the bus, 98 noisy kids on the bus.)

Concrete Activities:

See II. (e) - reverse the order. Begin with representation for 73 and have children identify the amount. Record the amount on the board. Remove ten, ask for new amount, and record. Continue through at least 23. Ask what the pattern is and what you are doing (counting backwards by tens). Repeat with fives and twos.

cuisenaire rods
or
Numeration bloc
or
Bean sticks
or
Tile strips

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS

III. (a) The child will read and write the numerals from 0 to 9.

See II. (c) activities	
Activities:	
Make 5" x 8" cards for each number from 0 to 9 with the word name for the number on one side, the numeral on the other side, and a pictorial representation under the numeral (eg. seven dots under the numeral 7). Attach the number of clothespins for the number at the top of each card.	5 x 8 cards felt mark pen clothespins
Have children remove clothespins from several cards, mix pins, then pull out the correct number of pins to put back on the cards.	
Have students copy the numeral and pictorial representation from the card.	
Use the black line masters from Mathematics Their Way to practice writing the numerals.	
Remember: most numerals face the opposite direction of most letters (alphabet) and children will often reverse the numerals. Make them aware of the problem but accept it as a normal problem of maturation.	

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

III. (a) Cont. The child will read and write the numerals from 0 to 9.

- also Workjobs I pp 140-179

Concrete operational pupils still having difficulty writing these numerals at 3rd grade probably need more tactile experiences with the numerals. Have sandpaper numerals for children to trace with their fingers (both with eyes open and again with eyes closed). Make large numerals on construction paper. Have the child glue yarn on the numeral. Give the child interesting ditto papers where pictures of sets of objects are followed iwth the numeral in dots for the student to trace. Have the child trace the felt numerals on Bell Bags while blindfolded.

Place plastic or wire numerals in a paper sack. Have children place hand in sack (without being able to see in) and determine which numeral is represented. (Can also be given pieces behind their back to feel and determine.)

ACTIVITIES FOR LEARNING THE COMPETENCY — MATERIALS

III. (b) The child will read and write the numerals from 10 to 999.

DO NOT HAVE CHILDREN WRITE NUMERALS UNTIL THEY CAN COUNT OUT THE NUMBER IN OBJECTS.

Activities:

Once children can write the digits from 0 through 9, the writing of multi-digit numerals is primarily a matter of spacing. Have children turn their lined papers side ways, make one digit in each space, and leave one or more spaces between numerals.

Graph paper is also helpful in helping pupils learn to properly space two digit numerals.

Peek-a-boo cards

Child is asked to read numeral made with 3 cards stacked - child may peek if can't read the numeral.

MAKE CARDS

$\boxed{100}$ (100 - 900)
$\boxed{20}$ (20 - 90 and 11 - 19)
$\boxed{9}$ (0 - 9)

Read number words (eg. three hundred seventy three) aloud for children to enter into the calculator for various reasons. See III (g) also.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

III. (c) The child will read and write symbols for "greater than," "less than," and "equal to."

See concrete operational second grade III. (c) for suggested activities.

Also introduce children to the "not equal" sign ($\neq$) by putting various amounts on a balance scale and recording whether it is equal ($=$) or not equal ($\neq$).

Competency/coordinated activities - 3rd grade
Concrete-operational

ACTIVITIES FOR LEARNING THE COMPETENCY

MATERIALS

III. (d) The child will read and write the equations for the basic facts with sums to 18 in addition and subtraction (numerals, operation signs, equal signs).

See concrete operational first and second grade III. (d) for suggested activities.

Game: Tuff (commercial)

Make dice of cubes as follows:

2-a) 4,5,6,7,8,9
2-b) 0,1,2,3,4,5
1-c) 10,11,12,13,14,15,
1-d) 13,14,15,16,17,18,
1-e) +, -, +, -, +, -
1-f) =, =, =, >, <, ≠,

eight 1" wooden
or sponge cubes
felt markers

egg timer

Pupils take turns rolling 2 addend dice (a or b), one sum die (c or d) and the operation and relationship dice (e and f). Child has set amount of time (eg. egg timer) to use dice to make a true math sentence from the dice as they came up. Score one point for each statement made.

Variations: Make several sets of the dice. Have students all roll at once. When egg timer ends, each person who made a true statement earns a point.

Scoring Variation: Count two points for persons rolling three A dice and a D dice; two points for an A and a D dice; adn 1 point for two B dice.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

III. (e) The child will begin writing number sentences expressing inequalities with numbers of nine and less (numerals, operations signs, greater than and less than signs.

See III. (d) - dice game.
Also see concrete operational first and second grade III. (e) activities.

ACTIVITIES FOR LEARNING THE COMPETENCY **MATERIALS**

III. (f) Pupil will write numerals from 0 - 9,999 when shown a manipulative model of the number.

Have children solve problems with concrete materials and then record what they have done.

Center Activity:

Have blank paper and base ten rubber stamps available. Pupils stamp on paper using stamps of their choice. They then write the numeral represented. (eg. A student marking 15 units, 12 longs, and 7 flats would record the number as 835.)

Base 10
rubber stamp
paper
pencil

Abacus and Calulator:

Have one student represent two different numbers on two different abaci. Have a second student enter each number into a calculator, pushing the plus key between the two numbers. Before second student pushes equal sign, first student combines the washers (beads) of both abaci onto one abacus, regrouping as needed. Students verify that calculator total and abacus total agree.

Competency/coordinated activities - 3rd grade
Concrete-operational

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

III. (g) Pupil will write numerals from 0 - 9,999 when from oral dictation.

Calculator/Activity Card

Print activity cards similar to the sample below:

cards
Calculator

add the following numbers:
Three thousand
four hundred twenty
one thousand twelve
three hundred seventy nine

4811

Have one child read the card aloud to a second child who enters the number into the calculator. They verify the correctness of the numeration by checking the sums.

See also: "Farmer Brown and the ______"
from Calculator workshop handout.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

III. (h) The child will read and write the equations for the basic multiplication and division facts (numerals, operation signs, equal signs).

See suggested activities for III. (d) and adapt these by adding multiplication and division signs and products to the card or dice games.

Multiplication

Introduce multiplication facts in 4 forms:

$$3 \text{ X } 7 = N \qquad \begin{array}{r} 7 \\ \text{x}3 \\ \hline N \end{array} \qquad 3\overset{7}{\overline{)N}} \qquad 3\,(7) = N$$

Division

Spend quite a lot of time helping children translate

$$21 \div 7 = N \quad \text{to} \quad 7\overset{N}{\overline{)21}}$$

(A recent study showed this is still confusing to teenagers and adults because of the reversed involved.)

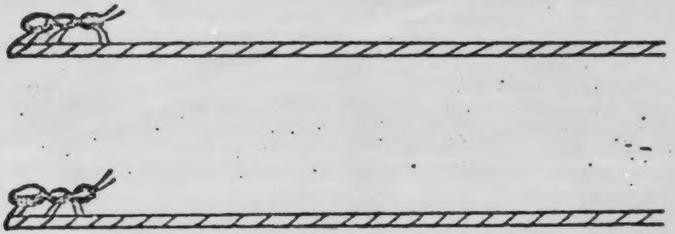
Use the terms "21 divided by seven" in both cases above. Tell children that some people read it as "how many 7's in 21" and other people read it as "7 guzenda 21" but the more correct form is to say "21 divided by 7" in both cases. Then model the correct terminology.

Conservation of Length

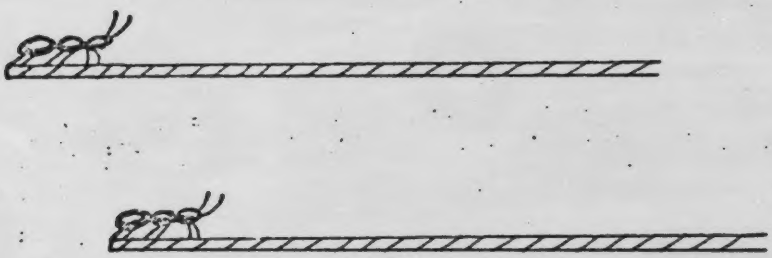
•*Purpose:* This activity will help the teacher evaluate a student's understanding of the invariance (conservation) of the linear qualities of concrete objects when they are moved about.

•*Materials:* Two equally long pieces of heavy cord.

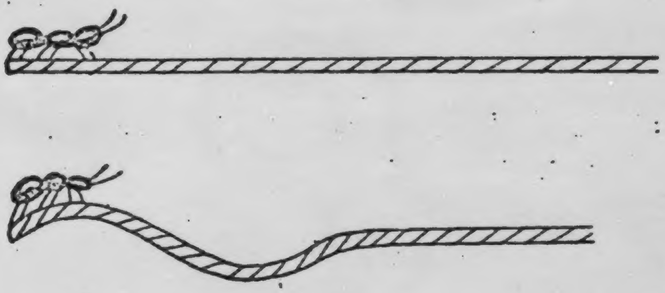
•*Procedure:* Present the following situation to the child: If two equally fast crawly-bugs were put on the end of these strings



and they both went the same speed, would they reach the end at the same time? How about these two strings? (Use the same strings.)



And these? (Again, use the same strings.)



Most children under six years old will not realize that the race would be a tie in the second and third cases even though they know you are using the same string. Around the age of six or seven, most children conserve length and give the correct response because they are able to reverse the process, mentally moving the strings into their original positions.

Children who do not conserve length should not be expected to measure with a ruler, since when they move the ruler it seems to get longer! The number line is of no help to a nonconserver of length. That child perceives the distance between four and five on the number line as being greater than the distance between one and two. Other, more perceptually sound models should be used in such cases.

Souviney, R. *et al*. *Mathmatters*. Goodyear, 1978.

Competency/coordinated activities - 3rd grade
Concrete-operational

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

IV. (a) Child will estimate length of real objects to nearest appropriate unit.

Whole Class Activity:

1) Give each student a white and an orange Cuisenaire Rod (or a unit and long of base ten blocks) and tell them that the small cube is one centimeter long and the long one is ten centimeters or one decimeter long.

2) Hold up a pencil, chalk board eraser, or some other classroom item. Instruct students to look at the object and estimate its length in centimeters (or white rods.) Have them write the estimate on scratch paper.

3) Ask someone to volunteer their estimate. Ask if anyone has a lower estimate. Keep going until lowest estimate is found. Record on board. Go back to highest number given and find if anyone has a higher estimate. Continue until highest estimate found. Record on board.

4) Make an honest estimate of the length of the object yourself (with no prior measuring.) Record this on the board as the teacher's estimate.

5) Have one student help you measure object. Stress that this is still an extimation (as are all measures) but it is a much more accurate estimate - within one-half centimeter of the object's length.

6) Continue activity with other objects. Ask students if they are getting closer in their estimates.

7) Introduce metric ruler. Continue as in 1 thru 4 above but now measure to nearest millimeter. If an object measures 137 mm write 13.7 cm because you ask students to extimate in centimeters.

Cuisenaire Rods or Base Ten Blocks

Various Classroom Objects

Metric Ruler or Meterstick

IV. (a) Continued - Child will estimate length of real objects to nearest appropriate unit.

8) After introducing this activity as a half hour full group activity, use it as a 5 minute time filler throughout the year, when you're waiting to be called to the cafeteria, etc.

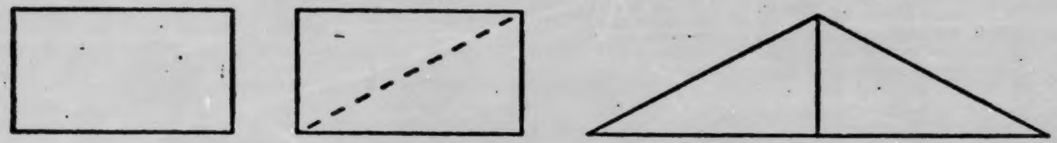
Conservation of Area

(Fraction Readiness)

•*Purpose:* This activity evaluates a student's understanding of the invariance of a concrete object's area when the object is divided into pieces.

•*Materials:* Two equal cardboard rectangles, one cut diagonally to form two equal triangles.

•***Procedure:*** Place the two triangles on the rectangle so that the triangles fit the surface of the rectangle. Once the child has agreed that these areas are equal (i.e. that the same amount of paint would be required to cover the triangles as to cover the rectangle) remove the triangles and arrange them as below.



Ask the child whether the two shapes are still the same size (i.e., whether they would require the same amount of paint to cover them). Most children under the age of seven will say that the second figure is larger. They are guided by their perception: the second figure has more pieces or simply appears larger. Between the ages of seven and eight, most children develop the notion that cutting and moving the pieces about does not change the total area. They are able to mentally reverse the procedure in order to justify their conclusions. Since the definition of fraction is often based on the area concept, formal work with fractions should be delayed until after understanding of area conservation is developed. Introducing informal investigations with fractional parts and fractional parts of sets of objects is appropriate only if conservation-of-number concepts have been mastered.

Souviney, R. et al. Mathmatters. Goodyear, 1978.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (b) Determine area of rectangles using one inch and one centimeter graph paper.	
Have numbered plastic and/or tag board rectangles (be sure to include squares) with dimensions in inches or centimeters. (DO NOT use rectangles with dimensions than cannot be traced onto the lines of the graph paper you are using.) Have students trace the rectangles onto the graph paper. Remind them to use the lines on the graph paper in tracing. Have them write the number of the rectangle on a separate paper and then list the width and length of the rectangle (using graph lines to measure.) The final step is to count the number of squares in the rectangle and list it beside the width and length. NOTE: Do not give students the formula for finding the area of a rectangle. After 2 or 3 activities similar to the above ask the class if they must count to find area. Let them discover L x W = A for themselves	Graph paper 1 inch or 1 cm Pencils Tagboard or plastic rectangles (measured in inches OR centimeters)

ACTIVITIES FOR LEARNING THE COMPETENCY

IV. (c) Find the volume of various shaped boxes with inch and centimeter cubes.

Set up a learning center with boxes of various sizes but with dimensions equal to inches or centimeters. (Use a box 3" x 2" x 4" NOT 2 7/8" x 1 3/4" x 4 1/8".)

Prepare activity cards requiring students to measure the box and choose the appropriate cubes (inch OR centimeter) to fill the box to capacity. They are to record measurements, cube size, and number of cubes needed to fill for each box. *

After each child has had experience with such a center, a follow-up center with new boxes would require students to measure boxes and then estimate the number of cubes needed to fill it. They can check their estimates by actually filling boxes with cubes.

See also: Sohns and Buffington, The Measurement Book, Enrich, 1977. and Primary Math Lab.

MATERIALS

Boxes of exact cm. or inch dimensions

Rulers

Inch cubes
Cm cubes

SAMPLE RECORDING SHEET

Recording Sheet

Letter of Box________
Height________in. or cm (circle one)
Length________in. or cm (circle one)
Width________in. or cm (circle one)

Number of cubes needed to fill________
size of cubes cubic in. or cm (circle one)

Conservation of Mass

•*Purpose:* This activity serves to evaluate a student's understanding of the invariance of the mass of a concrete object when its shape is transformed.

•*Materials:* Two equal amounts of clay.

•*Procedure:* Have the child hold the two pieces of clay and transfer clay from one to the other until both pieces are equal. Roll each piece into a ball and place them on the table in front of the student. Then roll one ball into a long sausage, leaving the other untouched. Ask the child whether both pieces are still the "same amount." Before age eight, most children will answer that the amount of clay has changed. Around the age of nine, however, most children are able to reverse the transformation mentally and to conclude that, since no clay was added or taken away, the mass has remained the same.

The introduction of mass-measurement concepts at the symbolic level to nonconservers of mass is not recommended. Furthermore, models such as the balance, which utilizes the mass concept to define number operations, should be avoided until mass conservation is attained. Considerable concrete experience with such devices, however, will optimize instruction at the primary level.

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Souviney, R. et al. Mathmatters. Goodyear, 1978.

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (d) Child will estimate mass of real objects to nearest appropriate unit.	
This activity almost requires a center or small group approach because each item must be handled before an estimate can be made. Set up a table with balance scale, mass pieces, and objects to be weighed. Ask students (via activity cards or oral instructions) to estimate the mass of each object and then verify their estimate by using the scale. Change the objects to be weighed at least once a week. If a kilogram spring scale is available, have students estimate their own mass in kgs and then use the scale to verify. *If no pan balance is available use a coat hanger with paper cups or bowls hung the same distance from each end (use three strings) and hang on a coat hook.	Mass pieces of various sizes (especially 1 gram plastic cubes) Balance scale* Junk box

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (e) Read time to the nearest minute.	
Obtain a digital clock and set it exactly with the traditional classroom clock. At various times throughout the day, call attention to the two clocks. For example: at 2:17 PM you might ask, "How do we say the time that is shown by the small hand just past the 2 and the large hand two small dots past the 3? Look at the other clock."	Classroom clock Digital clock
Many students will learn to read the round clock to the nearest minute from this activity alone because they will look back and forth at the two clocks many times on their own.	
See Kids Stuff Mathematics, P159. See The Measurement Book, p 179-198.	

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (f) Money: Determine change to be made from purchases of specific amounts.	
PLAY STORE	
Have students take turns being the cashier in a play store. Have other students pay for items with play money. Both customers and cashier must count the change.	Play objects to sell Price tags Play money Play cash register
LEARNING CENTER	
Have pictures of objects cut from catalogs mounted on activity cards. Mark price of each item at less than one dollar. For each picture make a second card showing the correct change if the item was purchased for one dollar. (These cards can have play coins affixed or be stamped with appropriate coin rubber stamps. Real money is even more appropriate.) Child's task is to match picture cards with correct change card.	Old catalogs Play money Coin rubber stamps 5 x 8 cards

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
IV. (g) Read a calendar	
Prepare calendar on bulletin board so that days of week and lines are permanent. Name of month and dates are to be changed each month. (Dates can be on cut out shapes that illustrate the month; e.g., hearts for February, shamrocks for March.	Classroom calendar
a) Have students take turns putting up new calendar each month.	
b) Have one student put up the current day's date each morning. Class then reads day and date.	

Set Inclusion

(Addition and Subtraction Readiness) •*Purpose:* This activity evaluates a student's understanding of the concept of a whole in relation to its parts.

•*Materials:* Two kinds of flowers (e.g., daisies and carnations)—seven of one (daisies) and three of the other (carnations).

•*Procedure:* Ask the child whether the daisies are flowers and whether the carnations are flowers. Then ask, "If you were to make bouquets, which would be larger, the bouquet of daisies or the bouquet of flowers?" Children under seven will generally indicate that the bouquet of daisies would be larger. They cannot simultaneously consider the *whole* set of flowers and at the same time distinguish between the *parts* (daisies and carnations). The child who is unable to deal with set inclusion is unable to reverse the process by breaking up a whole and realizing that a recombination of the parts is once again equivalant to the whole. This is analogous to a child's knowing that $3 + 5 = 8$ but not recognizing that $8 = 3 + 5$. Around the age of seven, children are able to consider both the whole and its parts simultaneously and can therefore solve the set-inclusion problem.

Before children have acquired a thorough understanding of set inclusion, it is not advisable to introduce the operation of addition and subtraction at the formal (symbolic) level. When addition is introduced before a child has gained competence with set inclusion, any learning may be rote in nature and true understanding of the concept may be delayed.

Souviney, R. et al. Mathmatters. Goodyear, 1978.

Transitivity

(Multiplication Readiness)

•*Purpose:* This activity evaluates a student's ability to recognize the equivalence of sets in an array rather than depending on one-to-one correspondence or counting methods to make that determination.

•*Materials:* Ten daisies, ten carnations, and ten vases.

•*Procedure:* Have the child put one daisy in each vase. Remove the daisies and repeat the procedure with the carnations. Ask whether he has used the same number of daisies as carnations. Then ask, "If all the flowers were put back in vases in the same manner as before, how many flowers would be in each vase?"

Children under six years old are generally unable to answer either question correctly. They are unable to conserve number, so when the daisies are removed and replaced by the carnations, they must re-count or place them in one-to-one correspondence in order to justify equality. These children are also unable to use the transitive property to assist them in solving the problem. That property could be described as follows:

Number of Daisies = Number of Vases

Number of Carnations = Number of Vases

Number of Daisies = Number of Carnations

Between six and seven years of age, however, children generally solve both problems readily, providing correct responses to the second question even when three, four, or more sets of flowers are considered.

Competence with the transitive property is of primary importance in the learning of multiplication skills. Transitivity and set-inclusion concepts develop in children at approximately the same time. This implies that while in the mathematics programs presently in use addition may be introduced a year early, introduction of multiplication may take place a year late! In any case, systematic introduction of symbolic multiplication should be delayed until after competence with transitivity is attained.

Souviney, R. et al. Mathmatters. Goodyear, 1978.

ACTIVITIES FOR LEARNING THE COMPETENCY **MATERIALS**

V. (a) Review all addition and subtraction facts.

See Bitter, et al. Addition and Subtraction Activities (entire book)

Card games

Prepare cards with addition and/or subtraction facts on one card and sum/difference on the second card. Use cards to play Old Maid, War, Concentration, Go Fish, etc. (Add one odd card for Old Maid.)

Blank cards

Felt marking pens

Slap: prepare about 10 cards of one color with sums or differences. With cards of a different color, prepare about 4 fact cards for each sum or difference. Make one card that reads "SLAP - COVER CARD"

1) Place answer cards randomly among 2-4 players with answers showing. Shuffle fact cards.

2) Place fact cards in a stack face up in the center of the group with the cover card hiding the top card.

3) Dealer removes cover card. Players try to be the first to slap the correct answer card.

4) Player who first slaps answer (bottom hand wins) takes the fact card from the deck and keeps it.

5) Person with most fact cards at end of game is winner.

ACTIVITIES FOR LEARNING THE COMPETENCY — MATERIALS

V. (a) Continued - Review all addition and subtraction facts.

Lotto games

Teacher prepared lotto cards can be laminated so that children can use crayons to mark out answer. Use BINGO or another 5 letter word at the top of the lotto card. Place sums or differences in the squares. Use facts on cards(shuffle and turn up one at a time), disks or balls are randomly drawn by a "caller". Only the sum or difference in the row called can be covered by the players.

5 x 6 cards

Felt marking pens

Alternative for addition

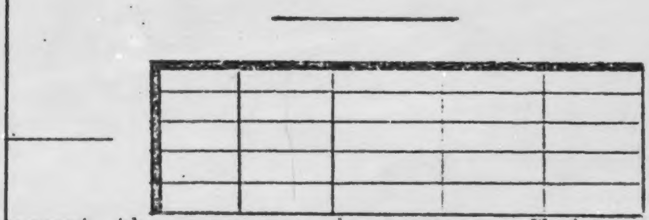
Have students fold scratch paper into 36ths. (Begin by folding in halves, then in thirds - both width and length or draw 36 squares.) Then write plus in the upper left square. Give students five single digits (e.g., 3, 5,7.8.9) and have them write one digit in each of the remaining squares on the top row. Numbers should be written in random order. Give the students five more numbers (e.g., 2,4,6,7,9) and have them put one in each of the left hand squares in random order.

Caller calls all possible sums from the digits given in random order. Students choose one square at intersection of two addends and write in the sum. (eg. sum = 14. Student can write at intersection of 7+7, 5+9, OR 6+8. But can only write it once each time 14 is called.)

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
V. (b) Use real objects to illustrate multiplication with products through 100.	
Pill Bottle Multiplication	
Label tops of pill bottles with a multiplication fact. Put loops of beads inside the bottle to match the fact written on the top. Example: 3x4 written on the top of the bottle with three loops each containing four pony beads inside. Be sure that the first digit in each fact tells how many loops and second digit tells how many beads in a loop. Hint: alternate two colors of beads for easier counting.	Plastic bottles Two colors of pony beads String Markers

Multiplication Cards:

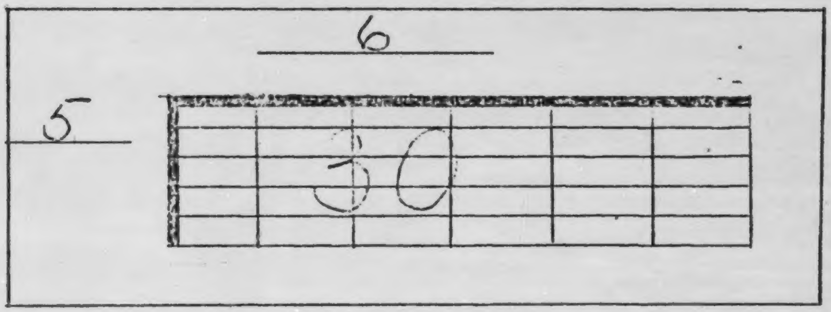
Laminate cards such as those illustrated below to have students begin study of multiplication facts (this also introduces children to the division algorithm).



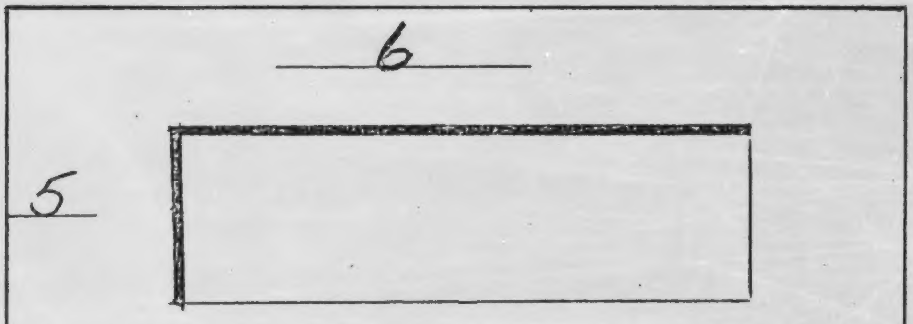
count the squares in a row. Write the number above. Count the squares in a column. Write the number on the line to the left. Count the total squares. Write the number over the squares.

V. (b) cont. Use real objects to illustrate multiplication with products through 100.

The child will use crayons to mark the card as follows:



After the children have used these cards for several weeks use cards such as the following for follow up.



Look at this rectangle. If the squares were filled in with the numbers shown how many little squares would there be? Write the number in the rectangle.

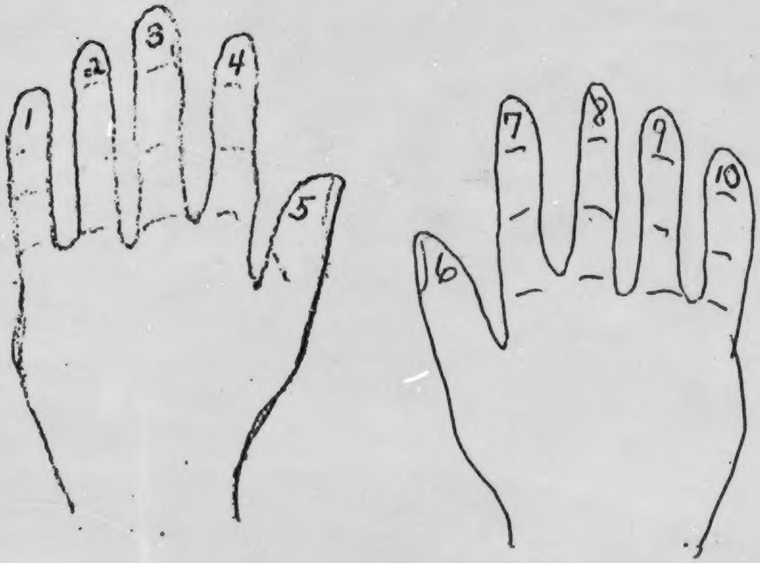
ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
V. (c) Recall multiplication facts with factors of 10, 5,2, or 9.	
See game of Buzz in instructional games. Use 10,5,2, and 9 as the buzz words.	
Have child count by tens, placing an orange rod or a long (from blocks) down at each number. Stop the child and ask the number represented by the rods. Emphasize that counting by tens is what we do when we use these rods. We can also count the rods and give a multiplication fact. If there are 5 rods we can say 5X10=50.	Cuisenaire Rods or Base ten blocks
Have children repeat the above while counting by 2's or 5's and placing red or yellow rods. Also can use groups of small objects either 2 or 5 at a time. Lead to the concept that counting by 2's or 5's is giving the multiples of those numbers and can be translated to knowing the multiplication facts using those numbers.	Cuisenaire Rods or small objects
NINES Use real objects to represent the multiplication facts for nines. Record the data on the board as follows: 1X9=09 2X9=18 3X9=27 4X9=36 5X9=45 6X9=54 7X9=63 8X9=72 9X9=81 10X9=90 11X9=99 optional at this point 12X9=108	pill bottles or other objects grouped in 9's.

V. (c) cont. Recall multiplication facts with factors of 10, 5,2, or 9.

Have children study this information. Ask "Do you see anything special about these facts?" If no one responds ask them to look for any patterns in the answers. Most classes note the pattern of one more ten and one less one first. Some, though, will note the fact that the two digits in the product always sum to nine. If both responses are made talk about them one at a time. Ask, "How could we use that pattern to help us memorize (or remember) these facts?" Let children respond.

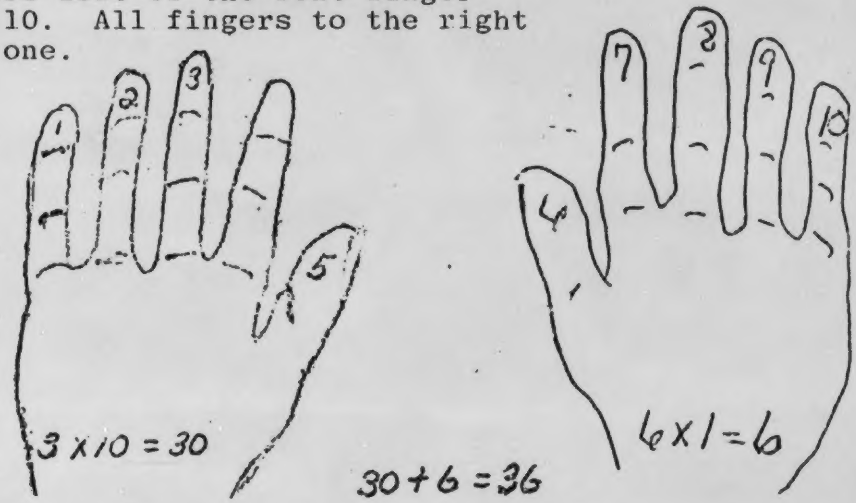
HAND TRICK

(To be used only after children have discovered some of the patterns of nines.) Have children label their fingers as follows: (Can use masking tape for children who cannot remember a fingers' number).



V. (c) cont. Recall multiplication facts with factors of 10, 5,2, or 9.

Give the children the fact 4 times 9. Tell them to bend down finger number 4. All fingers left of the bent finger count as 10. All fingers to the right count as one.



(Note to teachers: try this yourself for each fact. It always works. Do you know why? Can you encourage students to figure out why it works? Hint: 9=10-1).

Counting by 9's

Encourage children to think of 9 as 10-1. When adding nine they can mentally subtract one and add ten to the difference or add ten and subtract one from the sum.

In addition to counting by 9's beginning with zero, have children race adding nines beginning with non-multiples. (eg. Have one child begin with 517 and another begin with 627. Have them write the sums when adding nines (count by nines from that point):

V. (c) cont. Recall multiplication facts with factors of 10, 5,2, or 9.

517	627
526	636
535	645
544	654
553	663
562	672
571	681
580	690

Stop at the same time. Check for accuracy. See who went the furthest. Have class look at the patterns generated.

BEGINNING DIVISION

Purpose of Activity

The purpose of this activity is to study children's operational concept of equal division.

Materials Needed

A set of 18 counters.

Procedure

The child is asked to divide the counters "so that you and I have the same amount."

Levels of Performance

Stage 1. A child at this stage arbitrarily divides the counters by grabbing two handfuls and separating them into two piles. Asked if she and the interviewer each have the same, she judges by the space occupied by each pile.

Another Stage 1 child begins on a numerical basis, using a one-for-me–one-for-you approach. She divides the counters correctly in this manner, but when asked if the amounts are equal, she says there is more in one pile which happens to be spread out more. She then tries to make them the same by compressing the more spread out set.

"But are they the same?"

"No."

Stage 2. The child aged 5 to 6 divides the counters in pairs of "one for you and one for me" and then arranges the two sets into some geometrical pattern to determine if they are equal. For example,

• • •   • • •
• • •   • • •
• • •   • • •

Stage 3. This child, 6 to 7, divides the counters in the same manner as the Stage 2 child, but knows the results will be equal based on the one-to-one correspondence in the division. She does not have to judge on what the results look like; that is, in terms of the same geometric designs or the same spaces occupied.

Teaching Implications

Readiness for the operation of division appears at about the same time as for its inverse operation, multiplication, and readiness for multiplication appears at about the same time as that for addition.

As described by Piaget, "Additive and multiplicative compositions [which include division] . . . are . . . correlative, and the mastery of the one implies that of the other."[14] Thus more emphasis might be placed on integrating multiplication and division activities with addition and subtraction, rather than delaying multiplication and division for one or more years.

Copeland, R. W. *Math Activities for Children*. Merrill, 1979.

V. (d) Use real objects to illustrate division as the inverse of multiplication.

Pill bottles:

Ask children what fact would go on 3 x 4 = 12 pill bottle if you wanted to show division. (If multiplication facts are on lid in permanent ink, cover with masking tape showing the division facts.) Write 12 ÷ 3 = 4 and 12 ÷ 4 = 3 on lids.

Tell children we can use division to find out how many strings of beads there are or to find out how many beads are on each string if we already know one of these facts and the total number of beads.

Continue showing division with objects. Show that 12 ÷ 4 = N can mean we are going to make 4 groups of equal size (put objects into 4 groups, one at a time) or it could mean that each group will have 4 items (subtract out 4 objects at a time). Have children give related multiplication fact for each case.

MULTIPLICATION OF NUMBERS

Purpose of Activity

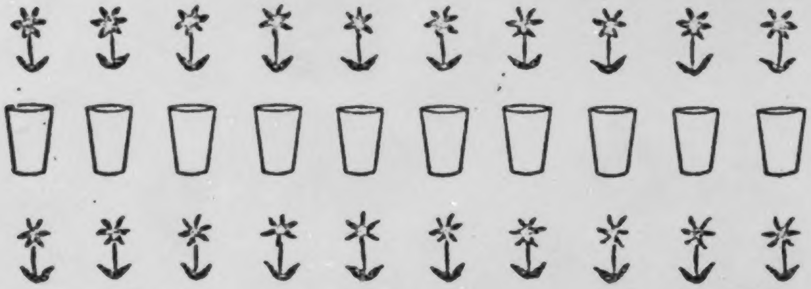
The purpose of this activity is to determine when children are able to solve problems involving multiplication of numbers.

The ability of children to make a matching or one-to-one correspondence between sets was studied in Activity 2-6 on conservation of number. From the psychological standpoint, a one-to-one correspondence is an implicit multiplication, for if two sets are in one-to-one correspondence, then the total number is twice or two times the number in each set.[13]

In this activity more than two sets will be considered in order to study children's ability to predict the total number as two, three, or four times the number in each set.

Materials Needed

Cardboard cutouts of 10 blue flowers, 10 yellow flowers, 10 vases.



Procedure

Ask the child to put a blue flower in each vase. Then remove the flowers and ask her to put a yellow flower in each vase. After these are removed, ask her if there are the same number of blue flowers as yellow flowers. And then, "If we put all the flowers in the vases with the same number in each vase, how many would there be in each vase?"

---

[13] Jean Piaget, *The Child's Conception of Number* (New York: W. W. Norton & Company, Inc., 1965), p. 204.

Levels of Performance

Stages 1 and 2. Between 5 and 6 years of age, the child can make a one-to-one correspondence of flowers to vases, but if one set of flowers is bunched up afterwards and the other set spread out, she thinks there are more flowers in the spread out set. She does not have the conservation of number concept.

Also, the child does not have the transitive idea:

If the number of

(blue flowers) = (vases)

and

(yellow flowers) = (vases)

then

(blue flowers) = (yellow flowers)

This transitive property of the equals relation may be expressed symbolically as

if $a = b$ and $b = c$, then $a = c$

The Stage 1 child cannot apply this logical idea.

At Stage 2, the child gradually succeeds by repeating the experiment or through the interviewer's questions such as "How many blue flowers were in the vases?" and "How many yellow flowers were in the vases?" But perceptual factors still influence her—as one set is spread out more, for example.

Stage 3. At 6 to 7 years of age, the child can immediately use the logic of the transitive property and conclude that since one blue flower went in each vase and one yellow flower went in each of the same vases, there must be the same number of blue flowers as yellow flowers.

The Stage 3 child can also answer the second question successfully: "If we put all the flowers in the vases with the same number in each vase, how many flowers are in each vase?"

If a third set of ten flowers is introduced and the question repeated, she can predict that each vase will hold three flowers.

If a set of straws is introduced with the restriction that only one flower will go in each straw, and the child is asked, "How many straws will we need for all the flowers?" she can predict how many. The Stage 2 child would have to go through the process of pairing straws and flowers to find the answer and then realize why the answer is what it is.

Teaching Implications

Multiplicative operations are seen to develop at the same time as additive operations in the mind of the child. Yet multiplication is not studied in comprehensive fashion until the third or fourth grades in most schools. This is probably because there are 100 addition facts to learn, and although they may not be easier, it takes time to learn them. Part of the problem may be beginning addition too soon and multiplication too late.

Copeland, R. W. Math Activities for Children. Merrill, 1979.

ACTIVITIES FOR LEARNING THE COMPETENCY | MATERIALS

VI. (a) Use real objects to add and subtract multi-digit numbers.

See the following pages from Open Math Resource Book illustrating multi-digit addition and subtraction with manipulative models. Do not use numerals when first introducing procedure.

NOTE: THE CHILD MUST DO THE MANIPULATING. THESE ARE NOT TO BE TEACHER DEMONSTRATIONS.

INTRODUCE:

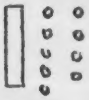
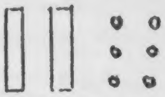
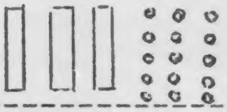
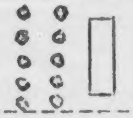
Try to use real situation such as there are 19 boys and girls present today in our room and 24 boys and girls in room x. If we all go to P.E. together how many boys and girls will be in P.E. Have students model the 19 and then model the 26. Have them join the sets. Then ask if they need to rebundle or trade (depending on material) to make the number easier to "read." Find regrouped number is 45.

After several informal situations (try to use real problems that involve regrouping right from the beginning) introduce more formal situations using a mat such as shown in Open Math or Mathmatters. (Mats can be made of butcher paper but pellon is very durable and can be kept for years.)

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

VI. (b) Record multi-digit addition and subtraction problems from manipulative model.

After several experiences such as those described in VI. (a), tell the children you are going to record what they are doing at the chalk board as they do it. For most children you would record directly into numerals as demonstrated in Open Math or below. For some children the pictorial model suggested by Mathmatters may be a necessary requirement. (Often a pictorial can be saved for use if a child has difficulty going directly from concrete to abstract.)

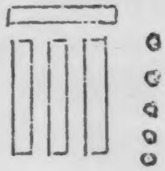
Teacher Says	Child Does	Teacher Write
"Represent the 19 boys and girls"		19
"Represent the 26 boys and girls		19 26
"Now join the two sets together"		19 +26
-OPTIONAL- (Teacher can write this or omit intermediate algorithm - depends on the group of children)	⟶	19 +26 15 30
"Do you need to do any regrouping (or trading)?"		

VI. (b) Cont. Record multi-digit addition and subtraction problems from a manipulative model.

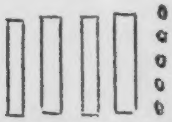
(If not using optional algorithm)

"Let's see - how will I record what you have done? I need to show that you traded 10 units for a 10."

"I will made a one in the 10's column to show the trade."

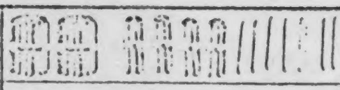
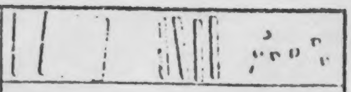
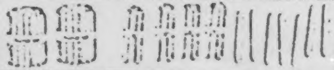
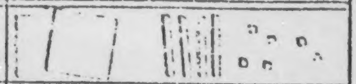
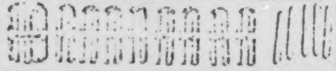
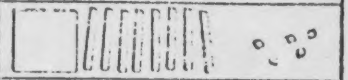
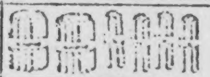
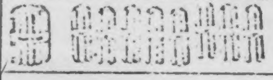
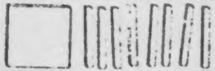
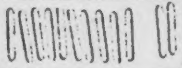
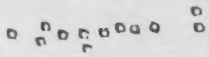
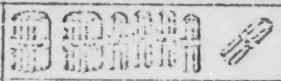
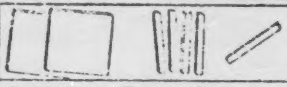
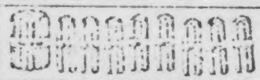
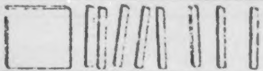
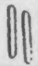
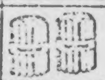
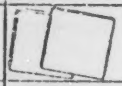
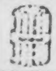
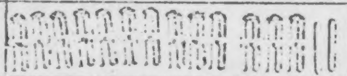
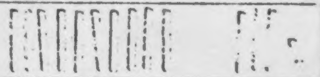
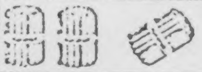
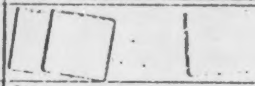
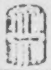
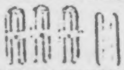
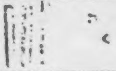
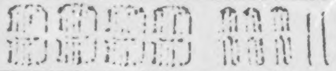
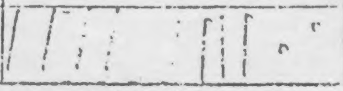


$$\begin{array}{r} 1 \\ 19 \\ +26 \\ \hline 5 \end{array}$$

Teacher Says	Child Does	Teacher Writes
"Now what was your total?"		$\begin{array}{r} 19 \\ +26 \\ \hline 45 \end{array}$

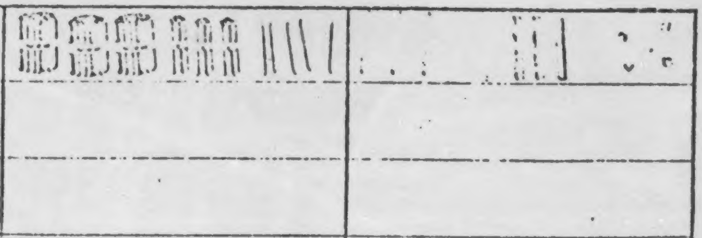
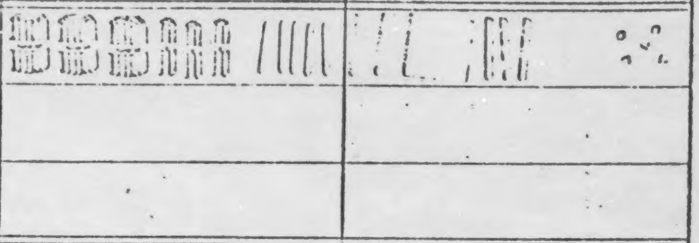
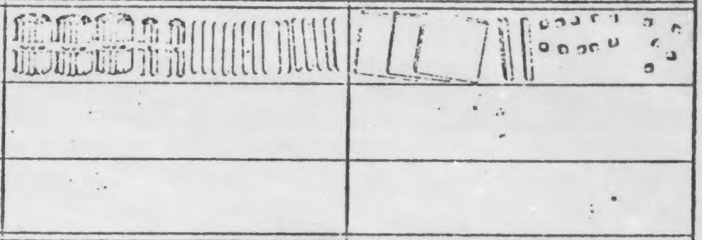
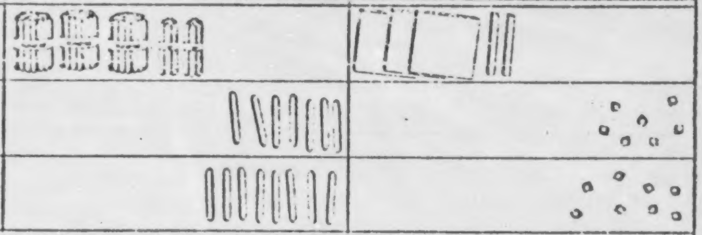
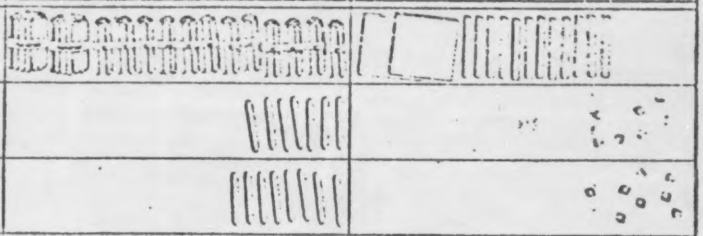
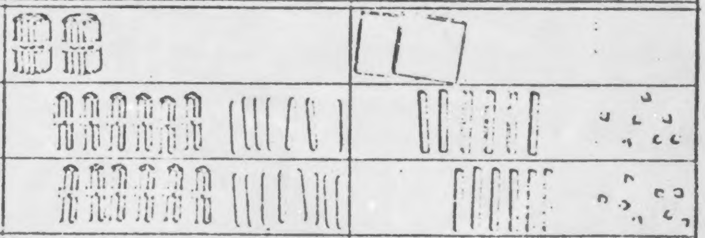
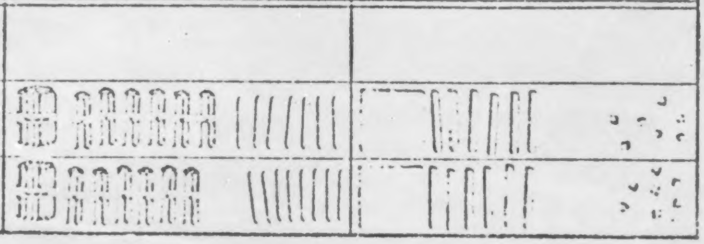
After 2 or 3 times of teacher recording from child's model, child should record from his/her own model.

ADDITION PROCEDURE

Represent the first addend.	247		
	+		
Represent the second addend.	247		
	+185		
Combine the ONEs; form a group of ten and a group of two	247		
	+185		
	2		
Trade the ten ONEs for one TEN.	$2\overset{1}{4}7$		
	+185		
	2		
Combine the TENs; form a group of ten and a group of three.	$2\overset{1}{4}7$		
	+185		
	32		
Trade the ten TENs for one HUNDRED.	$\overset{1}{2}\overset{1}{4}7$		
	+185		
	32		
Combine the HUNDREDS.	$\overset{1}{2}\overset{1}{4}7$		
	+185		
	432		

Knaupp, J. Open Math Resource Boo

SUBTRACTION PROCEDURE

Represent the minuend.	335	
	-	
Determine what the subtrahend will be.	335	
	- 167	
There are only five ONEs, so a TEN must be exchanged for ten ONEs. Now there are fifteen ONEs.	$3\overset{2}{\cancel{3}}{}^{1}5$	
	- 167	
Take away seven of the fifteen ONEs, leaving eight.	$3\overset{2}{\cancel{3}}{}^{1}5$	
	- 167	
	8	
There are now only two TENs, so a HUNDRED must be exchanged for ten TENs. Now there are twelve TENs.	$\overset{2}{\cancel{3}}\overset{12}{\cancel{3}}{}^{1}5$	
	- 167	
	8	
Take away six TENs, leaving six TENs.	$\overset{2}{\cancel{3}}\overset{12}{\cancel{3}}{}^{1}5$	
	- 167	
	68	
Take away one HUNDRED, leaving one HUNDRED.	$\overset{2}{\cancel{3}}\overset{12}{\cancel{3}}{}^{1}5$	
	- 167	
	168	

Knaupp, J. Open Math Resource

Because the concept of place value is lost when the two digits are thought of as separate entities, children understandably encounter difficulty when they must eventually learn to regroup. Suddenly, the two digits must be related in some manner and cannot be dealt with separately.

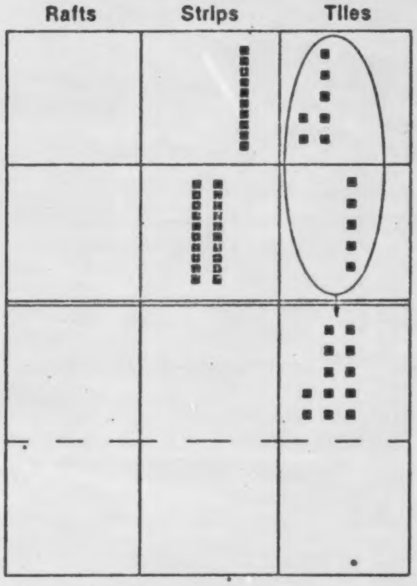
To avoid this confusion, two-digit addition with regrouping must be introduced simultaneously with nonregrouping situations. The concept should be presented in such a manner that both facets (regrouping and nonregrouping) are viewed as a natural and integral part of the process of addition.

Activity 2-G1.
(concrete)

•*Materials* Tiles, tile strips, Work Sheet 2-G1.

•***Procedure*** Students are given boards and asked to work problems that are given to them verbally. Again, place values are represented on the board by columns for single tiles, strips of ten, and rafts of one hundred.

When asked, for example, to add seventeen and twenty-five, students first combine all the single tiles (ones) and move them to the answer block.



Students are then asked how many individual tiles there are in all and whether this is the "best way" (see Activity 2-B1) to show twelve. They should respond that the best way would be by means of a tile strip and two single tiles. At this, the teacher points out that students must therefore trade ten individual tiles for a tile strip and move the tile strip into the appropriate column.

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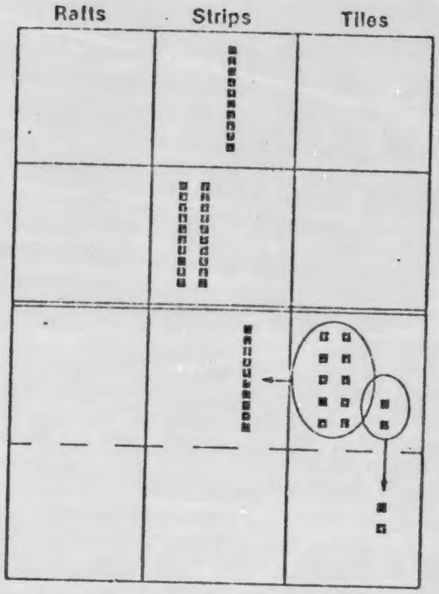
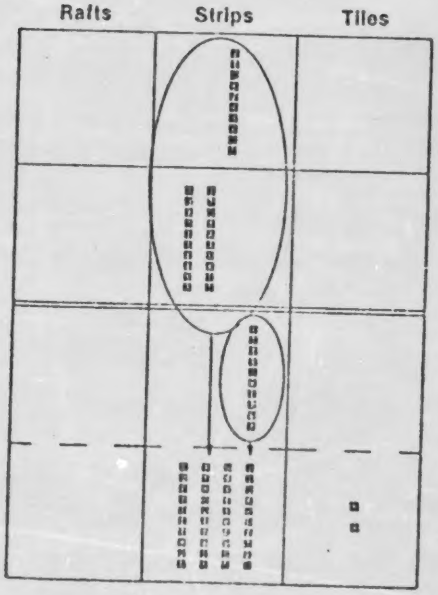

Rafts
Strips
Tiles

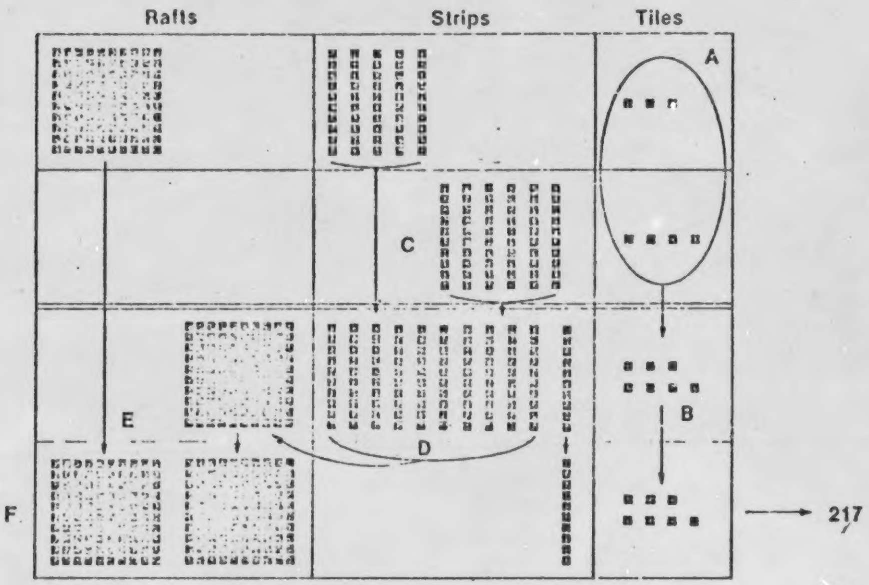
The same process is then repeated for the tile strip column.


Rafts
Strips
Tiles

Souviney, R. et al. Mathmatters. Goodyear, 1978.

The resulting group of tile strips (including the one compiled from the tiles place) is examined to see whether it, too, is the "best way" to show forty. Since in this case forty is already represented correctly, the problem is completed and the answer is forty-two.

Here is how the sum 153 + 64 would be handled:



All the single tiles are combined (A) and then checked to see whether the sum is represented in the "best way" (B). In this instance, it is represented in the best way, so we leave it alone. Next all the tile strips are combined (C). This time we must change the tile-strip group to the "best way" (D) by trading ten strips for a raft of one hundred. The raft is placed in its column (E) and with the other raft is brought down to the answer block where they are again checked for the "best way." Thus, the answer (F) is obtained.

Activity 2-G2.
(concrete/ representational)

•*Materials* Tiles, tile strips, addition organizer, pencils, Record Sheet 2-G2.

•*Procedure* Once again, give students an addition problem to be solved by means of addition boards, tiles, and tile strips. Ask them to use our representational notation in order to record the problem-solving process. In this way they experience both a concrete situation,

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Name____

ADDITION ORGANIZER

	Tile Rafts	Tile Strips	Tiles
+			

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Name ______________________________

1.

Rafts	Strips	Tiles

2.

Rafts	Strips	Tiles

3.

Rafts	Strips	Tiles

4.

Rafts	Strips	Tiles

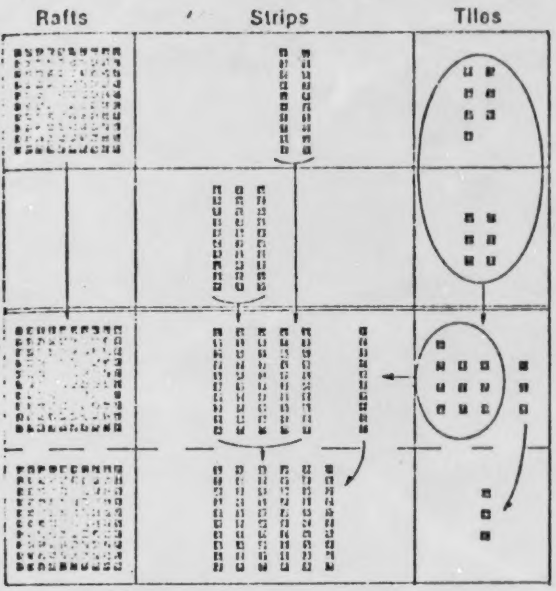
5.

Rafts	Strips	Tiles

6.

Rafts	Strips	Tiles

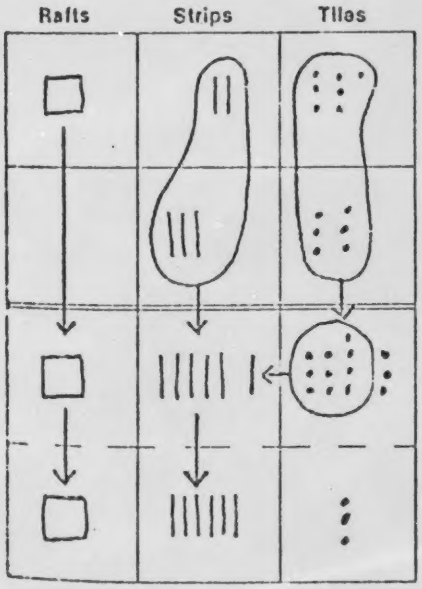
Souviney, R. et al. Mathmatters. Goodyear, 1978.


Rafts
Strips
Tiles

$$\begin{array}{r} 127 \\ +\ \underline{36} \end{array}$$

(Numbers are for teacher reference only.)

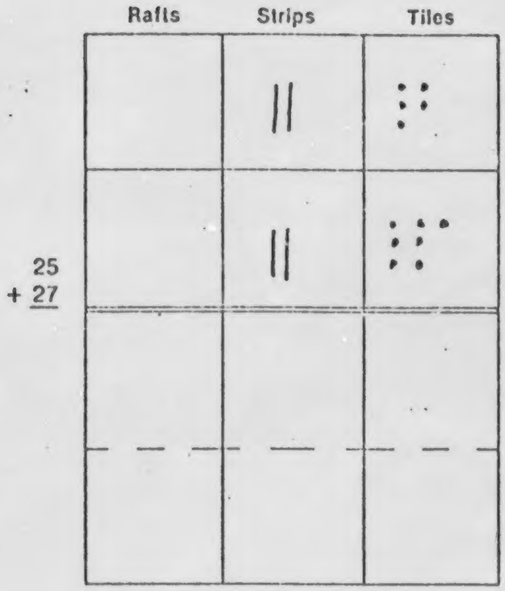
and the making of a representational record.

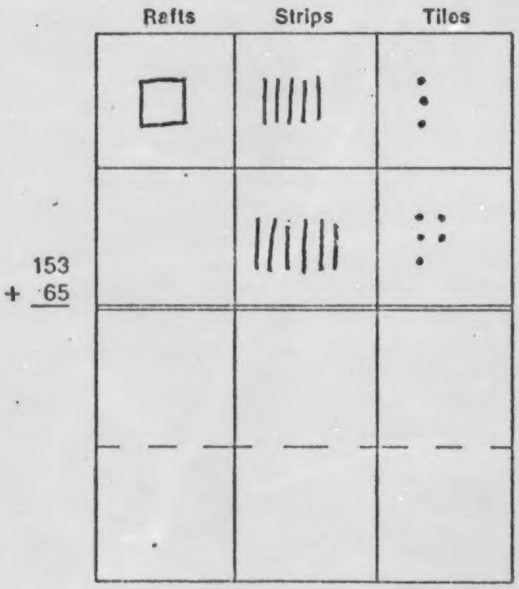

Rafts
Strips
Tiles

Souviney, R. et al. Mathmatters. Goodyear, 1978.

In contrast to the concrete model, representational notation allows the original addends to remain visible. After students have acquired extensive experience in using this notation as a recording process, the tile-strip model may be eliminated and all work done at the representational level.

Work sheets like the following may be developed (numerals are for teacher reference only; problems are given orally):





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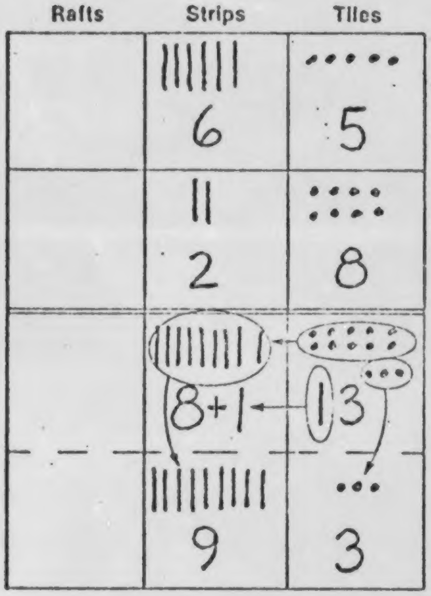
Activity 2-G3.
(representational/ symbolic)

•*Materials* Pencils, tiles, tile strips, Work sheets 2-G2 and 2-G3.

•*Procedure* Students are given representational problems and asked to make a symbolic record of their transactions on Record Sheet 2-G2.

When students have become familiar with the mechanics of symbolic notation, the representational schema is eliminated and problems are handled in an entirely symbolic manner (Work Sheet 2-G3). After computing the answer, students should write the entire problem next to the chart.



26
+ 92
118

100s	10s	1s
	2	6
	9	2
	11	8
1	1	8

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ADDITION

Name ____

Example:

	Hundreds	Tens	Ones
A.		3	4
+		2	8
		5	12
		6	2

$$\begin{array}{r} 34 \\ +\underline{28} \\ 62 \end{array}$$

	Hundreds	Tens	Ones
1.	1	2	6
+		9	2

	Hundreds	Tens	Ones
2.	1	2	7
+	2	1	1

	Hundreds	Tens	Ones
3.	5	2	9
+		7	8

	Hundreds	Tens	Ones
4.	2	9	0
+	3	7	8

	Hundreds	Tens	Ones
5.	7	0	6
+		9	8

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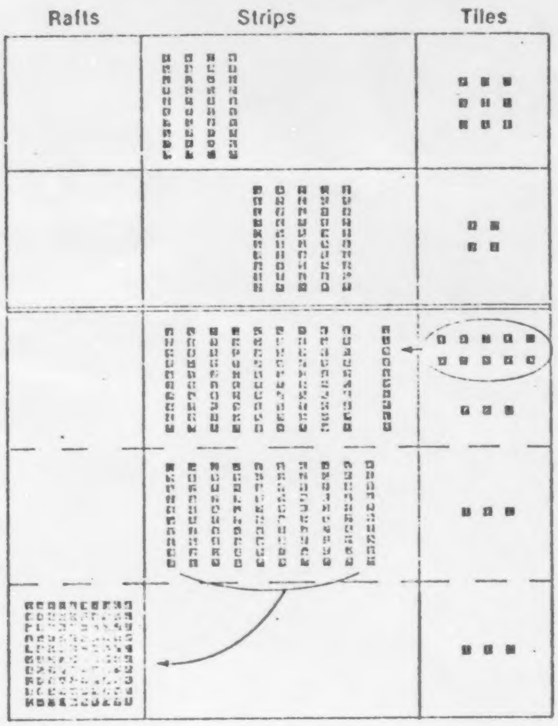
Ask them to validate these symbolic manipulations using tiles and tile strips.

$$\begin{array}{r} 49 \\ +\,54 \\ \hline 103 \end{array}$$



Students first complete the problems at the symbolic level, then validate their answers using concrete materials.

Activity 2-G4.
(symbolic)

•*Materials* Pencils, Work Sheet 2-G3.

•*Procedure* A fascinating phenomenon for children to explore, as well as a welcome alternative to the traditionally dull routine of drill and practice, is the palindrome, a word or number that reads the same forward and backward: pop, tot, toot, radar, 101, 2112, 23232, 111. Palindromes can also be complete sentences: Madam, I'm Adam.

A most interesting aspect of the numerical palindrome is the way in which it can be derived. Pick any number, reverse it, and add the two.

$$\begin{array}{r} 421 \\ +\,124 \\ \hline 545 \end{array}$$

The result is a palindrome!

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Sometimes it takes more than one step.

273
+ 372
645
+ 546 Reverse and Add
1191
+ 1911 Reverse and Add
3102
+ 2013 Reverse and Add
5115 Palindrome

This will always work. You can use any number and eventually you will reach a palindrome! A calculator might be handy if you try the number 89.

Concept H: Subtraction with Regrouping

Note: When students are working with multiple-digit numbers, subtraction with and without regrouping are introduced simultaneously as two facets of the same concept that should not be separated (see the introduction to Activity 2-G1).

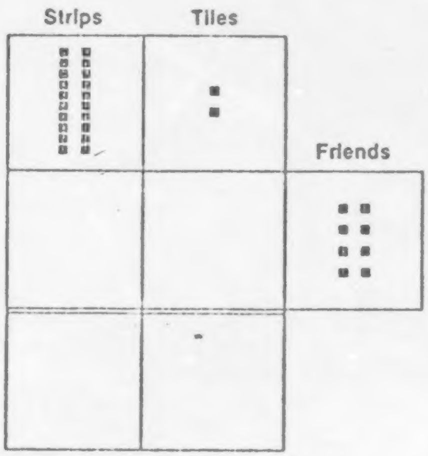
Activity 2-H1.
(concrete)

- *Materials* Tiles, tile strips, subtraction boards.
- *Procedure* Subtraction problems are set up in this manner:

Example 1.

(Numbers are for teacher reference only.)

22
− 8



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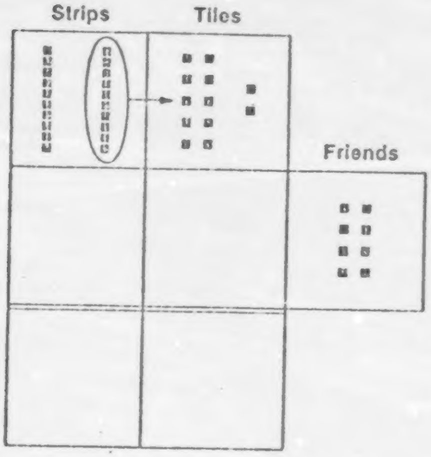
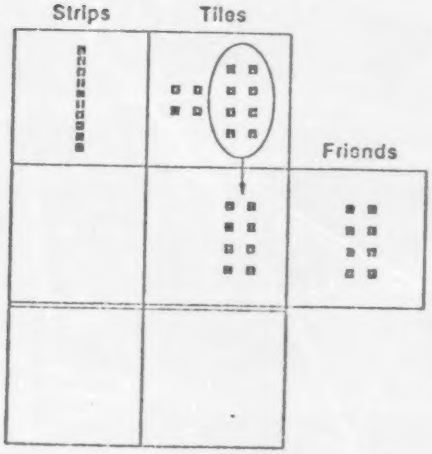
The tiles in the far-right box represent the "friends" used when developing the concept of subtraction (see Activity 2-F2). This should be made evident to the students.

Ask students whether they have enough single tiles to give each friend one. After they respond negatively, ask them where they could possibly get more. Encourage them to exchange a tile strip for ten individual tiles, which are then placed in the appropriate position.



Now are there enough tiles for each friend to have one?

The process may now be completed.



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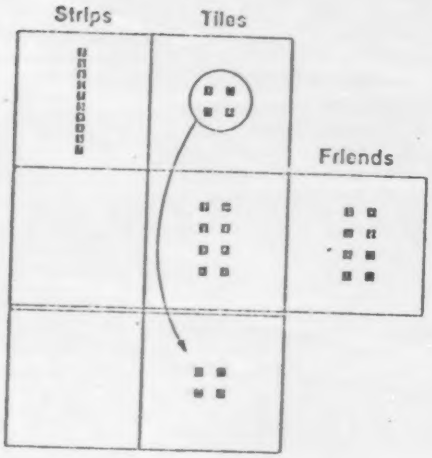
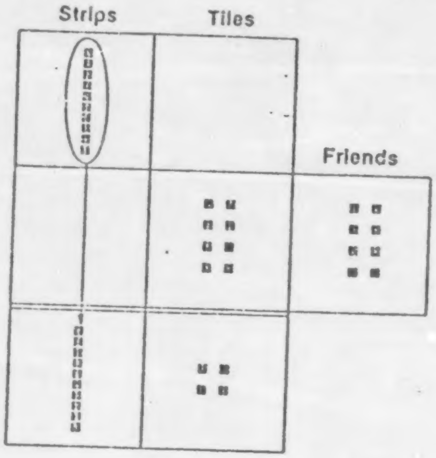
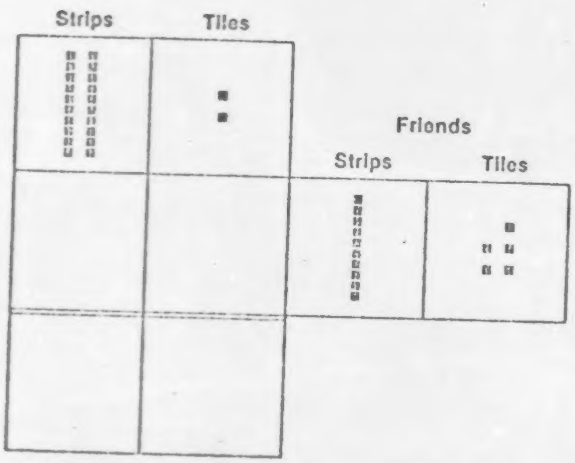
First, one tile for each friend is "taken away" from the original quantity. Next, the answer (first the ones place, then the tens) is brought down to its final position.


Strips
Tiles
Friends


Strips
Tiles
Friends

Example 2.


Strips
Tiles
Friends
Strips
Tiles

$$\begin{array}{r} 22 \\ -\underline{15} \end{array}$$

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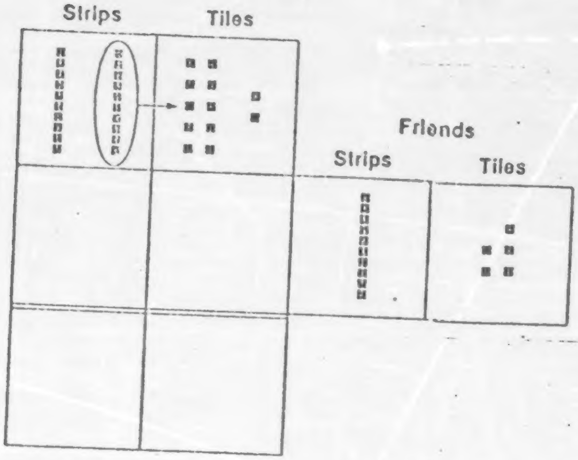
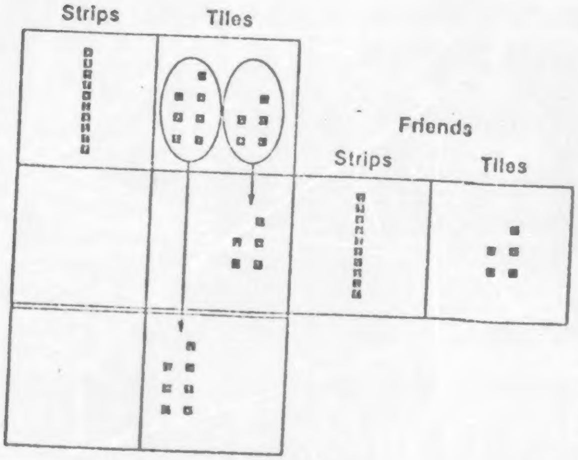
Step 1. Regroup tile strip to tiles place.


Strips
Tiles
Friends
Strips
Tiles

Step 2. Take away appropriate number of tiles (from the ones place) and bring the rest down to their final position.


Strips
Tiles
Friends
Strips
Tiles

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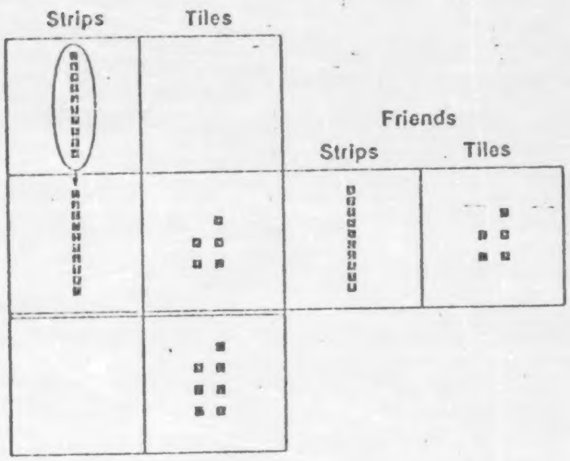
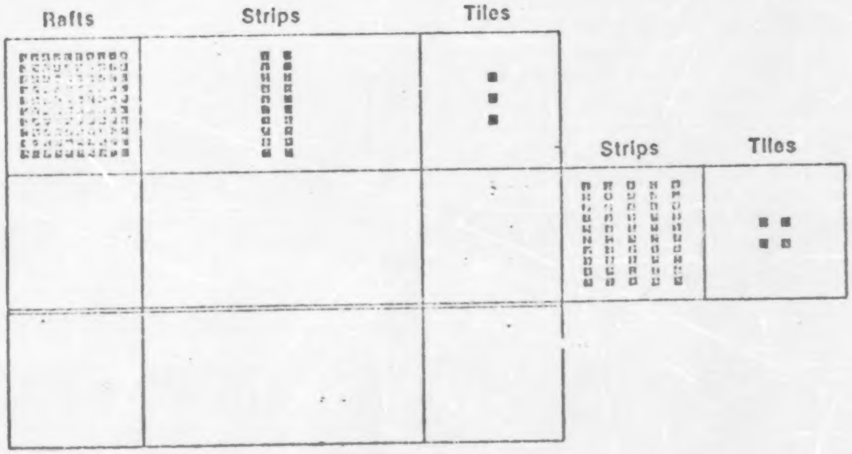
Step 3. Take away appropriate number of tile strips (from tens place) and bring any remaining strips down to the final-answer place (in this case, none).


Strips
Tiles
Friends
Strips
Tiles

Example 3.


Rafts
Strips
Tiles
Strips
Tiles

$$\begin{array}{r} 123 \\ -\underline{54} \end{array}$$

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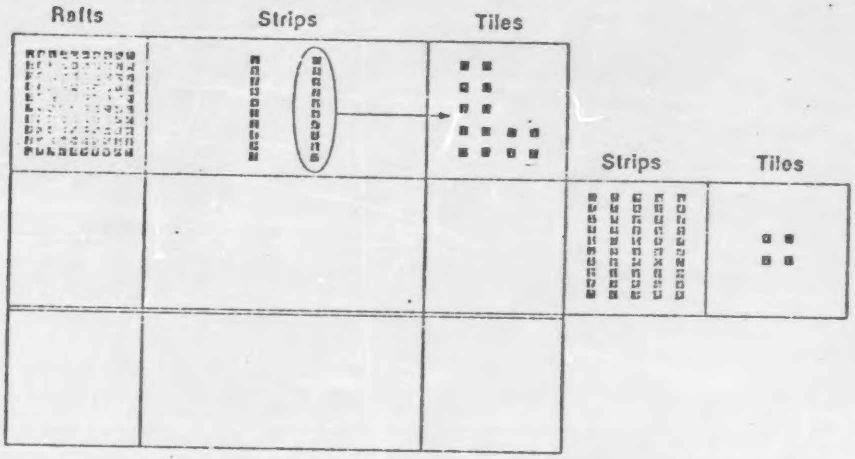
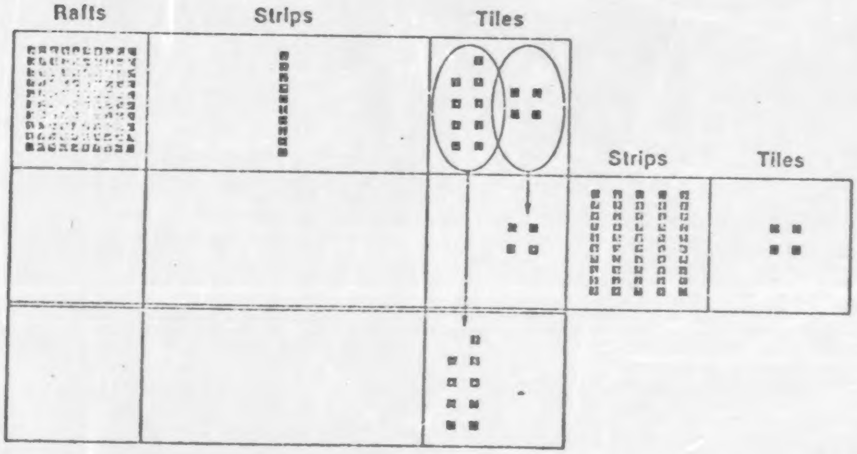
Step 1. Since there aren't enough loose tiles, regroup one tile strip and place tiles in ones column.


Rafts
Strips
Tiles
Strips
Tiles

Step 2. Subtract appropriate number of tiles and bring down remainder.


Rafts
Strips
Tiles
Strips
Tiles

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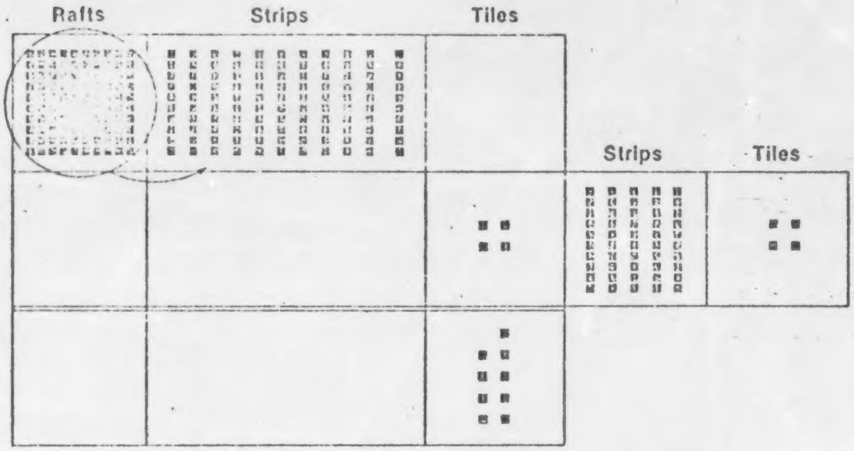
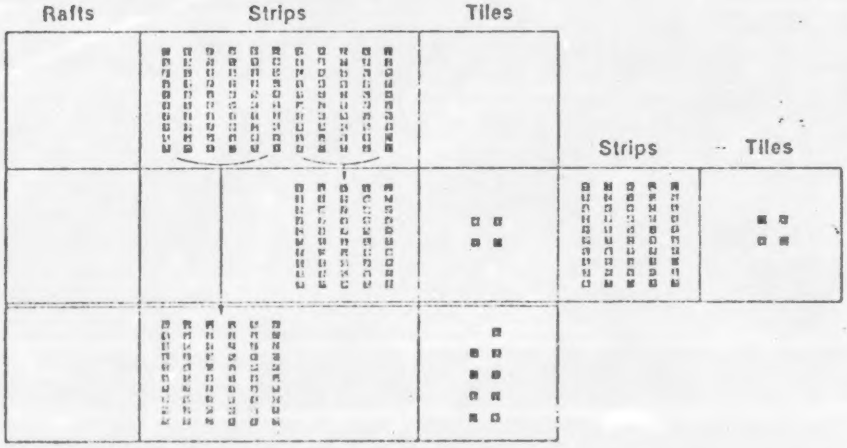
Step 3. Since there aren't enough tile strips, regroup one raft and place in tile-strip column.


Rafts
Strips
Tiles
Strips
Tiles

Step 4. Subtract appropriate number of tile strips and bring down remainder.


Rafts
Strips
Tiles
Strips
Tiles

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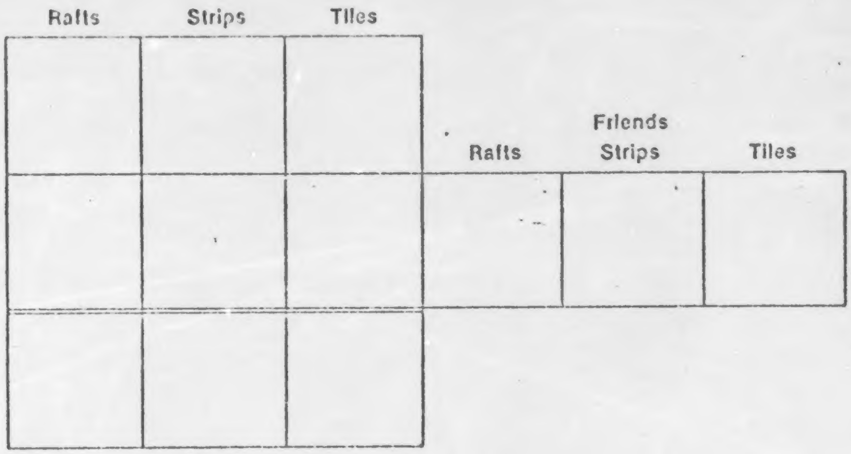
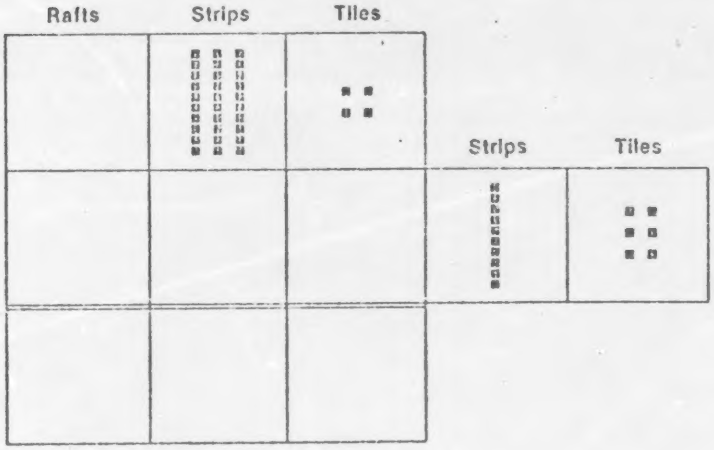
Activity 2-H2.
(concrete/ representational)

•*Materials* Tiles, tile strips, Record Sheet 2-H2.

•*Procedure* Repeat Activity 2-H1, but this time have students record their process using representational notation. Develop work sheets with the boards already drawn on them, i.e.,


Rafts
Strips
Tiles
Friends
Rafts
Strips
Tiles

For the problem 34 − 16, the concrete situation would look like this:


Rafts
Strips
Tiles
Strips
Tiles

and the representational record would look like this:

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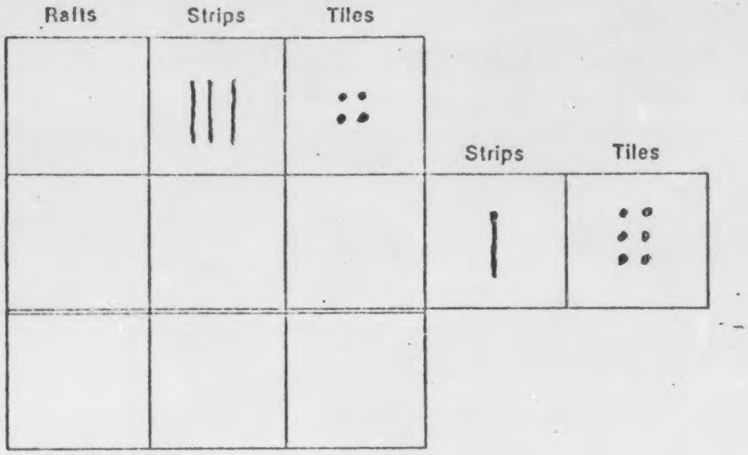
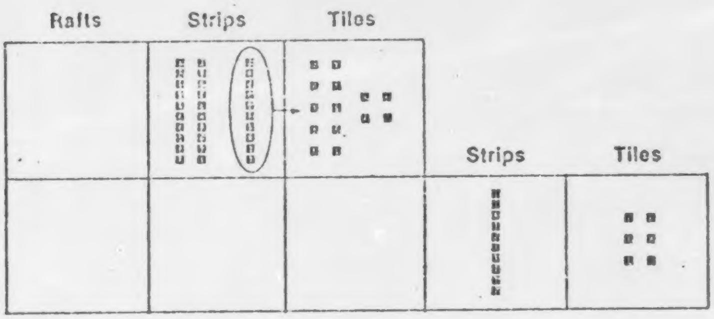
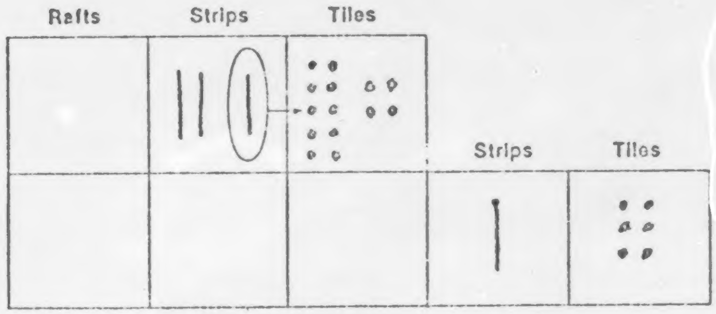

Rafts
Strips
Tiles
Strips
Tiles

First regroup tile strip to tiles place.


Rafts
Strips
Tiles
Strips
Tiles


Rafts
Strips
Tiles
Strips
Tiles

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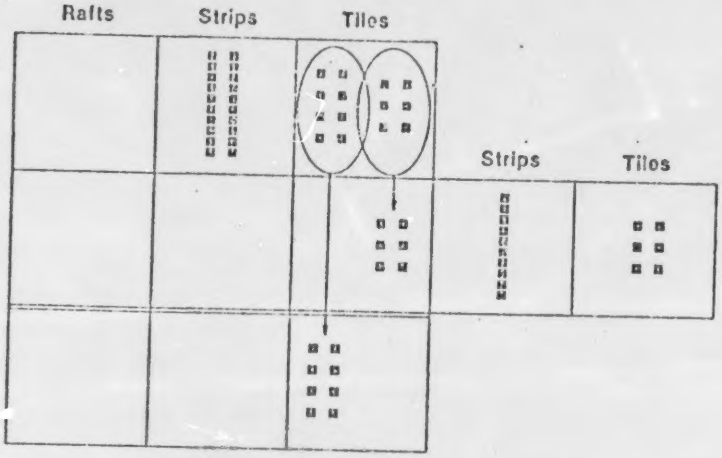
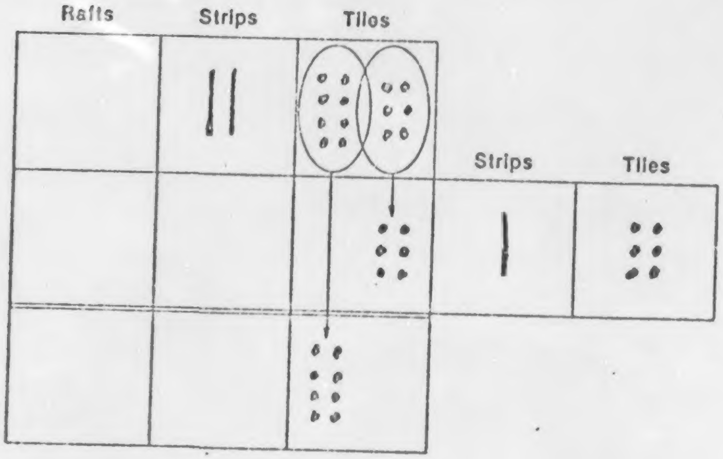
Next, subtract appropriate number of tiles and bring the answer down to the ones place.


Rafts
Strips
Tiles
Strips
Tiles


Rafts
Strips
Tiles
Strips
Tiles

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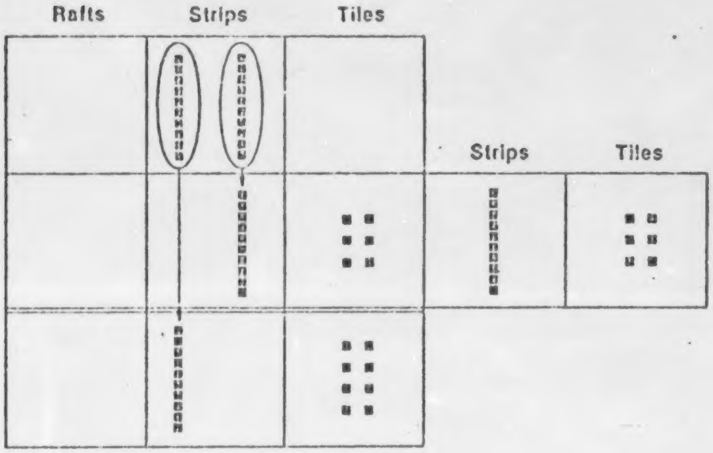
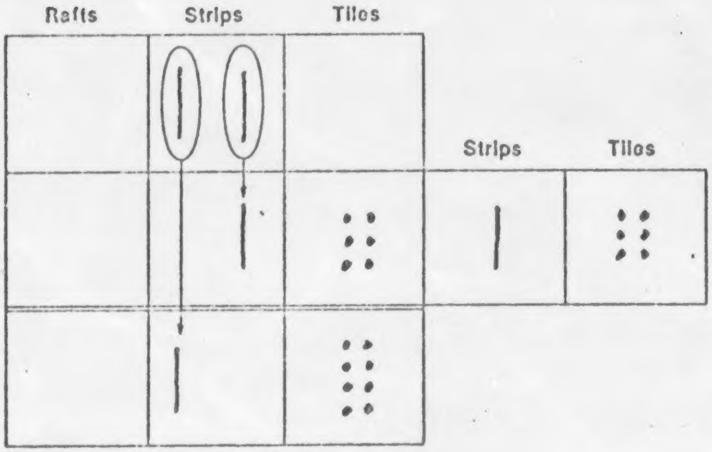
Finally, subtract the appropriate number of tile strips and bring the remaining answer down to the tens place. The problem has now been solved.


Rafts
Strips
Tiles
Strips
Tiles


Rafts
Strips
Tiles
Strips
Tiles

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SUBTRACTION

Name____

Rafts	Strips	Tiles

Friends

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Friends

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Friends

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Rafts	Strips	Tiles

Rafts	Strips	Tiles

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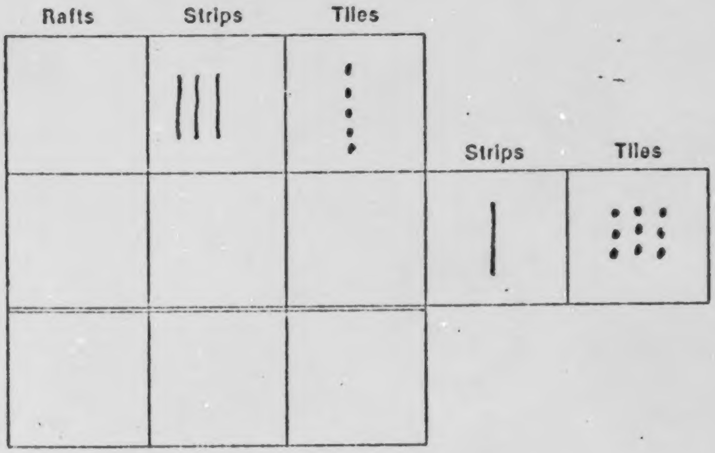
Activity 2-H3.
(representational/ symbolic)

•*Materials* Paper, tiles, tile strips, Record Sheet 2-H2.

•*Procedure* To familiarize students with symbolic notation for subtraction, have them use numerals to record their representational process on a separate sheet of paper.

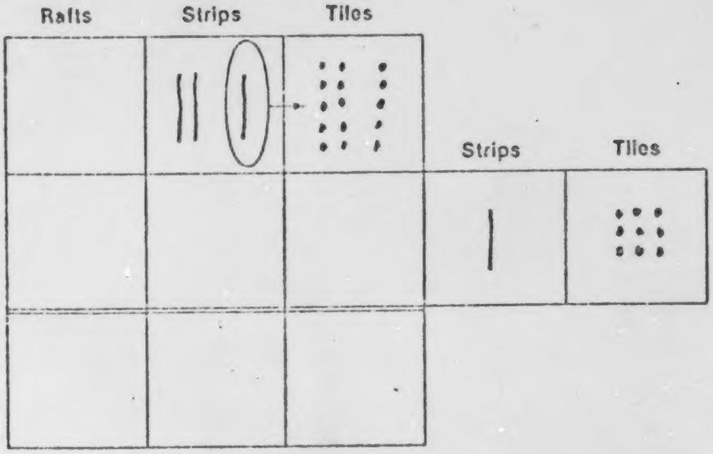
Original Problem:


Rafts
Strips
Tiles
Strips
Tiles

Record:

$$\begin{array}{rr} 3 & 5 \\ -1 & 9 \\ \hline \end{array}$$

Regroup:


Rafts
Strips
Tiles
Strips
Tiles

Record:

$$\begin{array}{rr} \overset{2}{\cancel{3}} & 15 \\ -1 & 9 \\ \hline \end{array}$$

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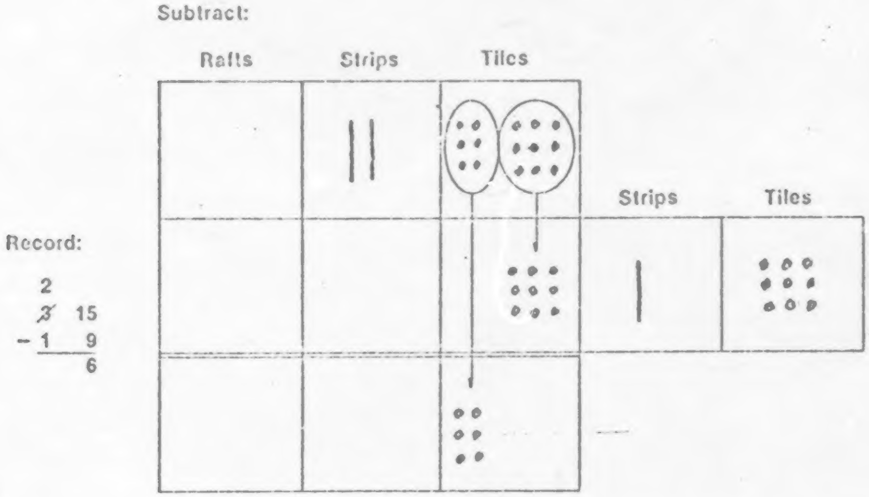
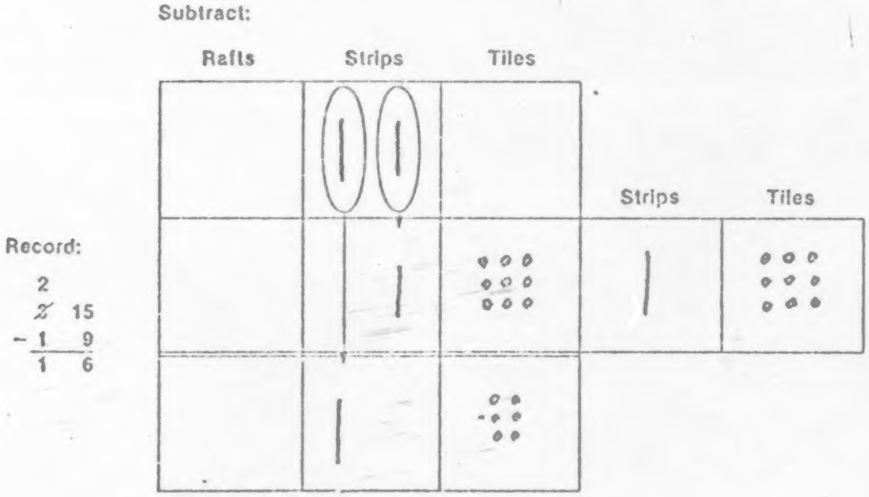
Subtract and bring down answer (ones column).

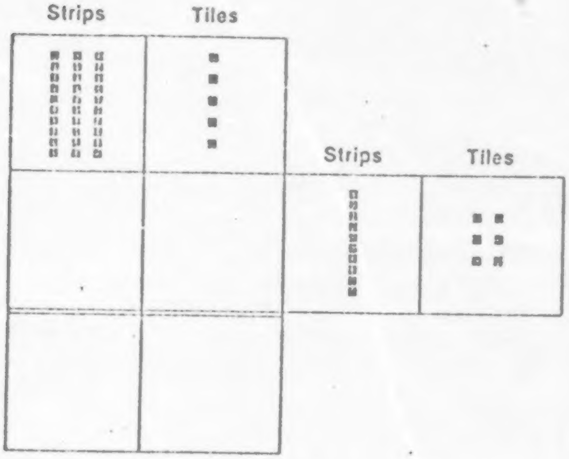

Subtract:
Rafts
Strips
Tiles
Strips
Tiles
Record:
2
~~3~~ 15
− 1 9
6

Subtract and bring down answer (tens column).

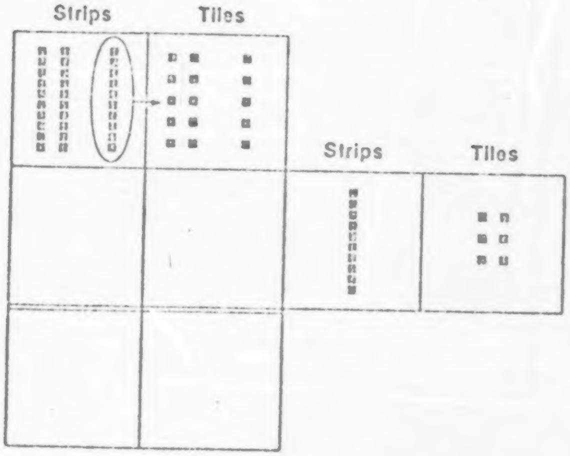

Subtract:
Rafts
Strips
Tiles
Strips
Tiles
Record:
2
~~3~~ 15
− 1 9
1 6

Souviney, R. et al.

After students have become familiar with the mechanics of symbolic notation, provide them with problems to be solved entirely at the symbolic level. Ask them to validate their answers by repeating the process with concrete materials or representational notation. For example, the problem 35 − 16 is given and the student finds an answer (19) at the symbolic level. He is then asked to validate his answer using tiles and tile strips. He sets up the problem,



regroups,



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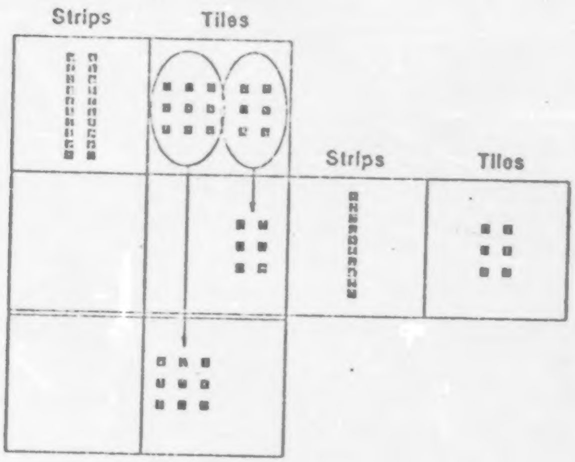
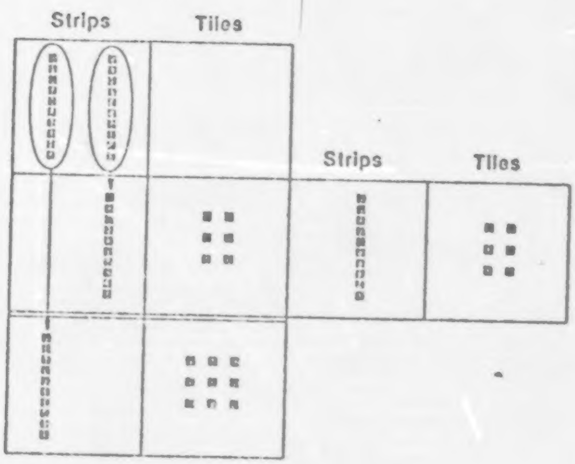
subtracts ones and brings down the answer.

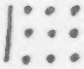

Strips
Tiles
Strips
Tiles

subtracts tens and brings down the answer.


Strips
Tiles
Strips
Tiles

Souviney, R. _et al._ _Mathmatters._ Goodyear, 1978.

The final answer is



i.e., 19—right!

Activity 2-H4.
(symbolic)

•*Materials* Pencils, Work Sheet 2-H4.

•*Procedure* Here is an interesting number property which can be used to spark students' interest in subtraction drills. Take any three-digit number (first and last digits must be different), reverse it, and subtract the smallest of the two from the largest. Next, reverse the difference and add. The answer will always be 1,089!

$$\begin{array}{r} 452 \\ -\ 254 \\ \hline 198 \\ +\ 891 \\ \hline 1{,}089 \end{array}$$

Here's another example:

$$\begin{array}{rl} 423 & \\ -\ 324 & \text{Reverse and Subtract} \\ \hline 099 & \\ \text{(In these cases you must include the zero.)}\ +\ 990 & \text{Reverse and Add} \\ \hline 1{,}089 & \text{Eureka!!!} \end{array}$$

This exercise provides a self-checking mechanism for a student's subtraction. If his final sum adds up to 1,089, he knows that his subtraction was correct; if he gets something else, he will know something went wrong.

Because the concept of place value is lost when the two digits are thought of as separate entities, children understandably encounter difficulty when they must eventually learn to regroup. Suddenly, the two digits must be related in some manner and cannot be dealt with separately.

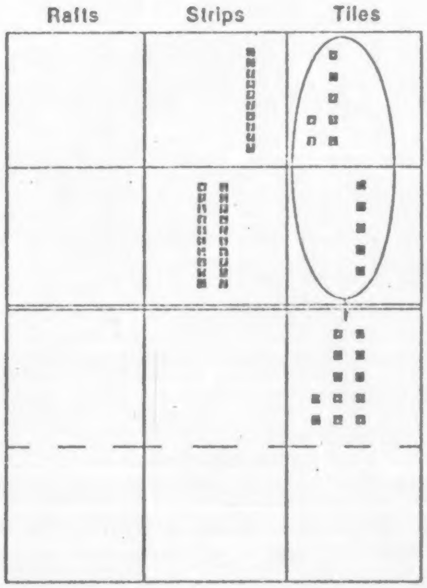
To avoid this confusion, two-digit addition with regrouping must be introduced simultaneously with nonregrouping situations. The concept should be presented in such a manner that both facets (regrouping and nonregrouping) are viewed as a natural and integral part of the process of addition.

Activity 2-G1.
(concrete)

•*Materials* Tiles, tile strips, Work Sheet 2-G1.

•***Procedure*** Students are given boards and asked to work problems that are given to them verbally. Again, place values are represented on the board by columns for single tiles, strips of ten, and rafts of one hundred.

When asked, for example, to add seventeen and twenty-five, students first combine all the single tiles (ones) and move them to the answer block.



Students are then asked how many individual tiles there are in all and whether this is the "best way" (see Activity 2-B1) to show twelve. They should respond that the best way would be by means of a tile strip and two single tiles. At this, the teacher points out that students must therefore trade ten individual tiles for a tile strip and move the tile strip into the appropriate column.

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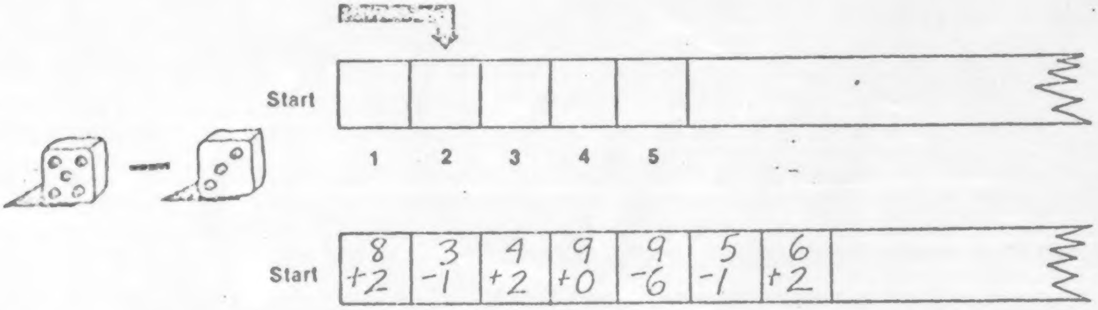
Activity 2-F3.
(symbolic)

•*Materials* Dice games.

•*Procedure* Develop games similar to the addition drills. This time, the two dice are rolled and the larger number placed in front of the smaller. The difference indicates the number of moves or the next number to move to. Games combining addition and subtraction should also be prepared.



Aside from providing valuable practice, these games allow students to enjoy what might otherwise prove to be a dull period of reinforcement in the teaching of mathematics.

Concept G: Two-Place Addition with Regrouping

In most mathematics sequences, one generally finds two-place addition without regrouping introduced before and apart from problems that involve the regrouping process. However, these two types of problems involve only one process, namely, addition of multiple-place numbers. Children do not view adding twenty-two and thirteen pieces of candy as being different from adding twenty-two and nineteen pieces. Furthermore, when addition without regrouping is introduced separately, children fail to recognize the place-value relationship in two-digit numbers. The problem

$$\begin{array}{r} 28 \\ +\ 41 \\ \hline \end{array}$$

becomes 8 + 1 and 2 + 4. (Ask a child which represents more in twenty-eight, the eight or the two.)

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ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

VI. (c) Solve multi-digit addition and subtraction problems using paper and pencil (using either an intermediate or standard algorithm).

An intermediate algorithm for multi digit addition is shown below:

$$\begin{array}{rl} 15 & \\ 18 & \\ \hline 13 & (5+8) \\ 20 & (10+10) \\ \hline 33 & \end{array}$$

An intermediate algorithm for multi digit subtraction is shown below:

$$\begin{array}{rcl} 33 & = & 20+13 \\ \underline{-18} & = & \underline{10+\ 8} \\ & & 10+\ 5 = 15 \end{array}$$

The standard algorithms are illustrated below:

$$\begin{array}{r} 1 \\ 15 \\ \underline{+18} \\ 33 \end{array} \qquad \begin{array}{r} \overset{2}{\cancel{3}}\overset{13}{\cancel{3}} \\ \underline{-18} \\ 15 \end{array}$$

NOTE: The 3 in the ones place is slashed out and 13 written above it. 2nd graders should show regrouping this wa They often do not "see" a 13 i a small one is inserted to the left of the 3. Also, when the have used the model first, this makes much more sense to them.

Either of these algorithms (or others the teacher may have developed) are appropriate for the child to demonstrate paper and pencil competencies.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

VII. (a) Students will prepare a bar graph from data they have collected and write at least one paragraph interpretting their results.

Examples:

Have children work in groups of 2 to 4. Choose data to be collected. (eg. Favorite TV shows, favorite soft drink, store where family shops for groceries, favorite movie star, month of birth, number of fillings in teeth, favorite cafeteria meal, etc.) Choose the audience to be surveyed. (eg. Your class, entire 3rd grade at your school, other grades or other classrooms, teachers, all adults on campus, parents, etc.) Assign each group a specific audience and data to be collected. Give them a specific time frame for collecting and recording their data.

After the group completes the data collecting step, allow them class time to organize their data and prepare bar graphs of the data for display.

Have each member of the group write a paragraph with their interpretation of the results.

For additional ideas on graphing, see:

OPEN MATH RESOURCE BOOK, pp. 148-150, 227-228
WORKJOBS, pp. 222-223
PRIMARY MATH LAB, pp. 57-60

AND

VII (a) First and Second grades.

PERCEPTUAL AND REPRESENTATIONAL SPACE

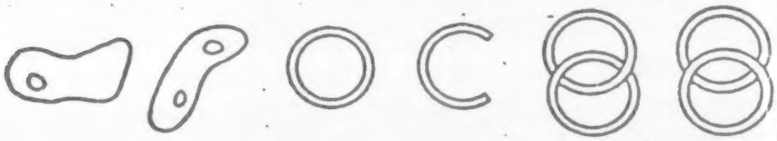
Purpose of Activity

Knowledge of space or the world around us develops in two ways—first, through what we perceive or what our senses tell us. Piaget refers to this concept as perceptual space. But our minds do not operate like cameras—photographing what our eyes see. Instead the mind must reconstruct or represent mentally such sensory data as obtained by touching or seeing. What the mind reconstructs is referred to as representational space, the second concept.

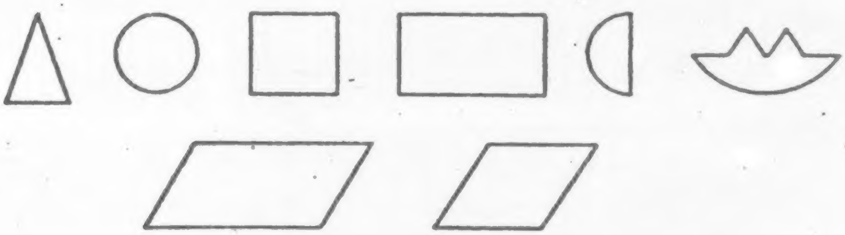
The purpose of this activity is to study children's ability to convert sensory or perceptual impressions of an object into mental representations of that object which, of course, is necessary for full understanding. What the mind "represents" may be and often is different from what is "seen" or "felt" by small children.

Materials Needed

1. A set of familiar objects such as a ball, comb, spoon, pencil.
2. A set of cardboard topological forms of a size convenient to handle.



3. A set of cardboard Euclidean shapes approximately 3 by 4 inches in size.



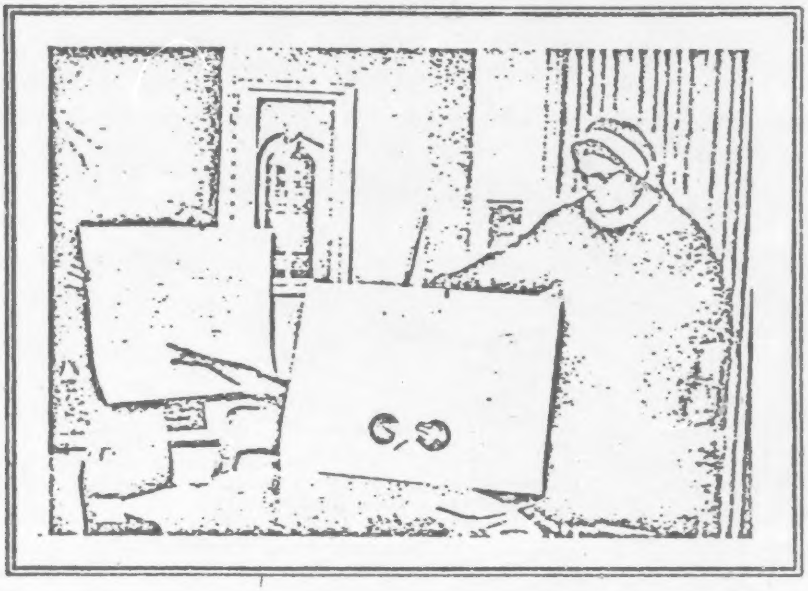
4. A cardboard screen with two holes in it for the child to put his hand through while handling objects he cannot see.

5. On a poster, pictures of objects the child is to handle (1–3).

Procedure

Ask the child to put his hands through the screen. Then, one at a time, give him objects to handle, asking him to point at the object on the poster.

For children 2 to 3 years old, use the familiar objects. For children 4 or older, begin with the topological figures. Piaget finds children able to make topological distinctions first—whether a figure is closed or open, has one or two holes, is intertwined or simply overlapping. It is more difficult to make Euclidean distinctions, such as noting the differences among triangles, squares, rectangles, parallelograms, and rhombi.



Levels of Performance

Stage 1. Familiar objects such as the comb are recognized, but children under 4 years of age are not coordinated in their action of handling the object. Coordination, of course, is necessary to construct a mental representation of the object.

Exploration is passive. No distinction is made between such Euclidean figures as the circle and square (since both are closed figures). Toward the end of Stage 1, distinctions are made between the topological figures such as whether the figure is open ⋐ or closed ◎.

Stage 2. Usually, children from 4 to 6 years of age can distinguish between curved Euclidean shapes and those with straight sides, but no distinctions are made between such rectilinear figures as squares and rectangles. Neither are the curvilinear figures, such as circles and ellipses, differentiated from each other.

Explorations of the figures is more methodical, however; such shapes as the triangle and square begin to be differentiated by number of angles.

Stage 3. Beginning at approximately age 6, children are operational as far as basic Euclidean shapes and even more complex figures such as a cross are concerned. Account is now taken of order and distance between points on the figure.

Exploration is methodical. Furthermore, although it begins with perceptual activities as in the other stages, this stage now uses an operational method of *grouping* the elements perceived, such as corners or angles and the distances between them. This grouping is dependent on beginning at a fixed point on the figure, tracing the outline and being able to reverse the process, tracing back to the starting point, mentally grouping those sensory impressions into a coordinated whole at the representational level. This, children in the first stages cannot do.

Teaching Implications

Teachers need to know the stages through which children go in developing the ability to consider geometric ideas. Many teachers may also not be aware of the fact that understanding space is not just a perceptual activity. For example, teaching what a triangle is by the "show-and-tell" method does not at all ensure that the child can mentally represent the idea of a triangle and hence understand it.

Perceptual activities are the starting point but are not sufficient. Mental coordinating actions of these sensory impressions must take place. These actions must often be based on physical or motor action, such as tracing and retracing a figure with one's fingers. For some children the necessary mental coordination may not occur until 9 or 10 years of age.

Also, the task of picking a figure from a collection that the child has handled but not seen is less difficult than having to represent or draw the figure. However, children should also have activities in representing figures by drawing them when they are able. If motor coordination is poor, the child may be asked to make the figures out of matchsticks.

Copeland, R. W. Math Activities for Children. Merrill, 1979.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

VIII. (a) Construct polyhedra from various materials.

(Polyhedra - solids formed from surface faces.)

Suggested materials:

- straws & clay
- D Stix
- Tinker toys
- Paper or tagboard

Activity

Flatten several boxes (odd shapes if possible) by cutting on one or more seams or folds. Ask children to guess what shape the boxes were originally.

DRAWING (COPYING) GEOMETRICAL FIGURES

Purpose of Activity

This activity is to explore children's ability to copy basic geometrical figures.

Materials Needed

1. A card with figures as shown on the next page.[3]
2. A pencil.

Procedure

Ask the child to draw a man from memory both to put him at his ease and to get some idea of his drawing ability. Then ask him to make a copy of each of the figures on the card. These figures make distinctions between topological and Euclidean concepts of space.

Levels of Performance

Stage 0. At 2½ to 3 years of age, drawings are pure scribbles.

Stage 1A. At 3 to 4 years of age, the scribbles vary according to the model being copied. Open shapes such as Models 20 and 21 can be distinguished from closed shapes such as Models 4, 5, and 6.

Stage 1B. The drawings of these children, also 3 to 4 years of age, Piaget refers to as the first real drawings bearing some resemblance to the figures being copied. Definite distinctions are made between open and closed figures, but the closed figures are not distinguishable from each other. A circle, triangle, and square may all be drawn as an irregular closed figure.

Stage 2. Children aged 4 to 6 show progressive differentiation between the basic shapes, such as circles, squares, and triangles, in their drawings. Curved shapes are distinguished from straight-sided

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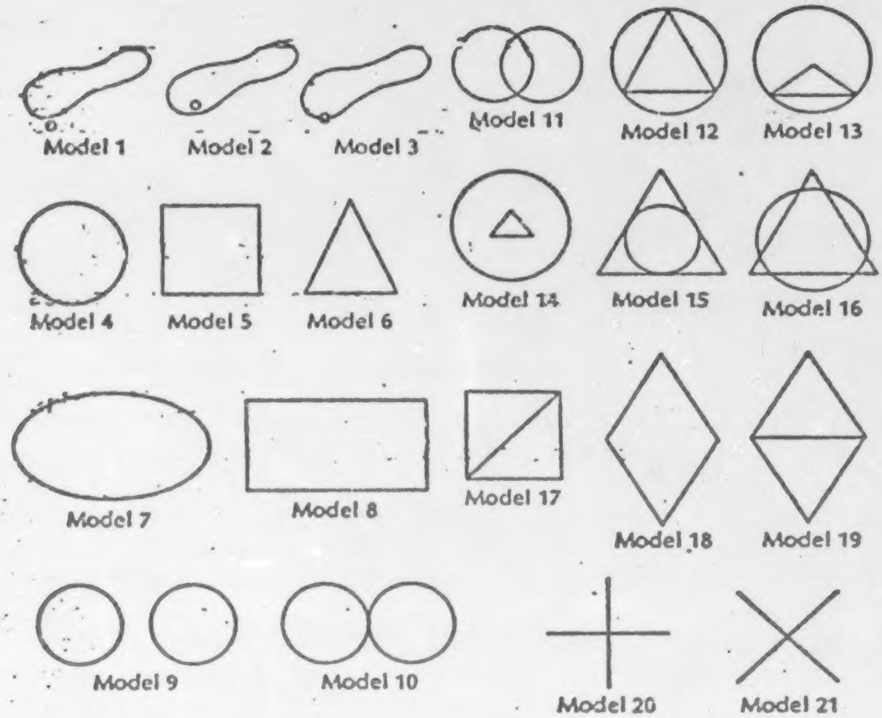
[3] Jean Piaget and Barbel Inhelder, *The Child's Conception of Space* (New York: W. W. Norton & Company, Inc., 1967), p. 54.


Model 1
Model 2
Model 3
Model 11
Model 12
Model 13
Model 4
Model 5
Model 6
Model 14
Model 15
Model 16
Model 7
Model 8
Model 17
Model 18
Model 19
Model 9
Model 10
Model 20
Model 21

ones, but little distinction is made at first between the curves or between the straight-sided figures.

A square and rectangle may appear to be the same. Gradually, differentiation is made in terms of number of angles. A triangle, for example, may be different from a square; a circle, different from an ellipse. The two crosses, Models 20 and 21, are drawn differently, indicating the recognition of difference between vertical, horizontal, and oblique. Circumscribed figures such as Models 13, 14, and 15 are drawn correctly as to shape, but the points of contact are not properly represented; that is, as touching, not touching or overlapping.

Stage 2B. The rhombus, Model 18, is now drawn correctly. Circumscribed figures such as Models 12, 13, and 15 (but not 16) are gradually mastered.

Stage 3. Children at this stage, 6 to 7 years of age, are able to draw all of the models correctly.

Teaching Implications

This activity provides a basis for determining in a clearly defined way what progress children are making in their ability to realize, as well as copy, basic spatial distinctions.

Children first make topological distinctions between figures. They distinguish "open" figures (Models 20 and 21) from "closed" figures, such as Models 4, 5, and 6. They are also able to make the topological distinctions in Models 1, 2, and 3: They can copy the small circle "outside," "inside," and "on the other figure." Some children, not functioning at the topological level, cannot even do this. In drawing a human figure, they may draw an eye outside the head or legs not attached to the body.

Later, children make Euclidean distinctions which show correctly the sides or angles; the length of sides; the size of angles; and vertical, horizontal, and oblique concepts. Thus they differentiate between such Euclidean shapes as squares, parallelograms, and trapezoids.

Most children will be ready at first-grade level to learn the basic shapes.

Copeland, R. W. Math Activities for Children. Merrill, 1979.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

VIII. (b) Use pattern blocks and mirrors to create a tessalating design.

See Open Math for introductory pattern block activities.

Make a center for 3 or 4 students with a large supply of pattern blocks and 2 mirrors per child. Show them that they can hold the mirrors at angles by their designs to determine if they tessalate (repeat continuously).

Have children find examples of tessalating designs in their environment (eg. tiles, some wall paper, some carpets, some fabrics, bee hives, etc.)

Also use pattern blocks and designs to establish lines of symmetry.

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

VIII. (c) Create and outline designs with tan grams for their peers.

Begin by introducing 3 or 4 easy shapes for children to find. (on the first one or two shapes include some interior lines.) (Use marcy cook's tangram mini center to introduce.)

Challenge pupils to create their own designs. Have them outline the design and add it to the center for other children to solve.

Have two children work with a screen between them. Have one child describe a tangram design in fron of him to the second child who must reproduce it from the oral description.

EXTENSION: After solving a tangram design have children use the small triangle to mark off the entire design. How many small triangles does the design contain. (16 if all pieces used.)

IX. (a) Explore number patterns involving both addition and subtraction.

See Mathmatters pages 254 - 275

Whats My Rule? (whole group game)

Leader (teacher or child) puts number function on the board with at least four numbers

eg.

?

5	8
7	10
3	6
12	15
9	

and the results of applying a rule. Then a 5th # is placed on the right and the group is asked to raise their hand if they think they know what and how the rule would affect that member. (In the example given the child would say the next number is 12.) The first child called upon who gives the correct answer is the "ANSWER MAN OR ANSWER WOMAN." That person chooses a new number to go in the left hand column and is the one who says whether guessing classmates have a correct answer. (You may want to give the answer person 1 or 2 additional examples to solve to be sure rule is understood.) Have various answer persons explain rule in their own words to the class.

Have a number pattern center with numerous laminated activity cards with number patterns to be completed and/or extended. (eg. 15,22,29,___,43,___,___,___) Have blank cards for students to create patterns for their peers. (They must have the correct answers to file.)

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

IX. (b) Use colored rods to make stair cases of odd and even numbers.

Method One:

Demonstrate and ask children to make the following staircases: red, purple, dark green, brown, and orange; and white, light green, yellow, black, and blue. Ask them to talk about what they have made. How big are the steps? (Red on both staircases.) How could the steps be the same when different colors were used? If white were 1, what would the numbers on these staircases be? (2,4,6,8,and 10 and 1,3,5,7, and 9). What do we think of when we see those two groups of numbers? etc. (try to solicit idea of odd and even numbers from the group).

Method Two:

Give children the rods and the rule that 1 white = 1. Ask them to make a staircase of all even numbered rods and all odd numbered rods. Ask questions about what they see.

Method Three:

Ask children to make their own staircases with steps of red. When finished compare and see if they are the same (some will probably start with white and some with red - if so, go to questions in method one). If all students have the same staircase, challenge them to find another way to make a staircase with steps of red. Then ask series of questions such as those in method one.

IX. (c) Explore non-linear patterns via pattern blocks.

See Pattern Blocks Activities B pp. 65-83

FRACTIONS

Purpose of Activity

The purpose is to explore children's understanding of the quantitative relations involved in dividing a whole into equal parts.

Materials Needed

1. Paper cut into various shapes, such as circles, squares, and rectangles.
2. Six dolls.

Procedure

Show the child a circular piece of paper and two dolls and ask, "If this were a cake, could you cut it so that each doll would get the same amount? Can you show me with this pencil how the cake should be cut."

Vary the procedure with other shapes, such as a square and a rectangle.

Also use the same procedure with three dolls, then six dolls.

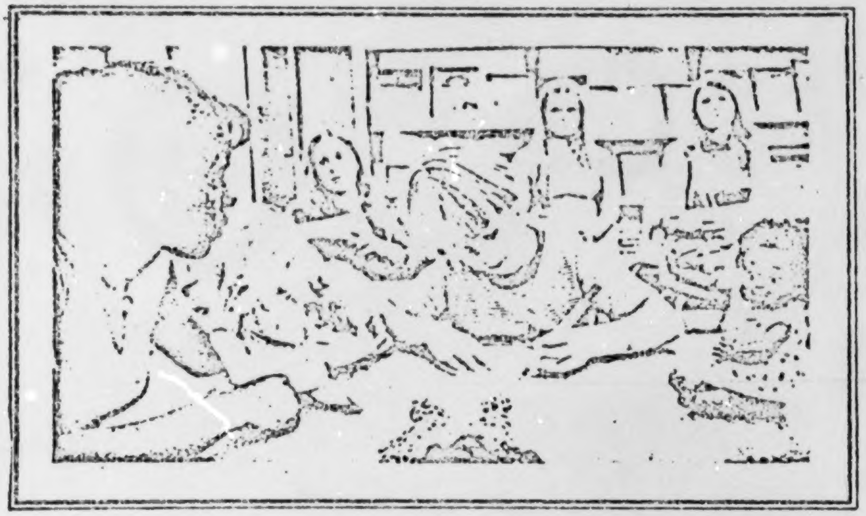
Levels of Performance

Stage 1. In children under 4 years of age, there is no plan and the divisions are not equal. The child is unable to relate the part to the whole once the cake is cut. The parts are now less or more than the original cake.

Stage 2. At Stage 2A, 4 to 6 years, children can divide small cakes into two equal parts, but not into three equal parts.

At Stages 2B and 3A, 6 to 7 years of age, children can divide into three equal parts, but it is a trial-and-error process; for example, drawing a line and then erasing it to redraw the line in a different position. These children do understand conservation of the whole—that once the cake has been cut it is still the same amount of cake.

Stage 3. At Stage 3B, around 10 years of age, children can with



Dividing a "cake" for three dolls.

some assurance tackle the problem of dividing the cake into six equal parts. They may, for example, first divide it into three equal parts and then divide each of the three into two equal parts.

Teaching Implications

Children are often introduced to fractions by being "shown" something divided and "told" the name of each part. Then they are given pictures divided into parts and asked to name the parts.

Children should do the dividing as well as naming the resulting fractional part. As this activity demonstrates, it is not as easy as one might think.

Copeland, R. W. Math Activities for Children. Merrill, 1979.

Competency/coordinated activities - 3rd grade
Concrete-operational

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
X. (a) Identify one-half, one-third, one-fourth, one-sixth, and one-eighth of such things as a cup of water and of the basic geometric shapes.	
Have children use measuring cups to experiment with fractions. Using the ½ cup and the cup, pour various ½ cups (water, rice, sand, etc.) into the cup. How does the cup look when you have put in ½ cup? How does it look when you put in 2/2 cups? Do two half cups always equal the full one cup? Continue with 1/3 and 1/4 cups.	Measuring cups (dry measure type) sand water rice, etc.
PRIMARY MATH LAB: pp. 63-71 (cooking)	
Use felt fraction kit and flannel boards to allow children to explore congruent fractional parts of a whole.	flannel board felt fractions - cir felt fractions - squ
Make note that notation is not being introduced - only concrete experiences of language.	

(Copy same activities as 2nd grade but use examples 1/8cup, 4/8cup, and 8/8 cup. Continue with all sizes.)

To introduce notation you might call attention to the markings on the container and how we read it. Only do this with children who are able to deal with the oral language of fractions.

Competency/coordinated activities - 3rd grade
Concrete-operational

ACTIVITIES FOR LEARNING THE COMPETENCY	MATERIALS
X. (b) Handle and count fractional models (eg. egg cartons or fraction bars) to amounts larger than one whole (eg. 5/3).	
Have each child paint their own set of egg carton fractions which you have cut. Establish a color pattern and display it. All fractions of the same size must be the same color.	egg cartons (12 or 30 egg size) knife or cutting board 4 colors - tempra paint
(Note: Use only halfs, fourths, thirds and wholes. Make both models of $\frac{1}{2}$. If using the 30 egg carton with 24 as a whole, make both models of $\frac{1}{4}$ also.)	
Hand a child a $\frac{1}{2}$ (egg carton or fraction bar) and ask him/her what she/he has. Hand him/her a second $\frac{1}{2}$ and repeat the question (answer 2/2). Hand him/her a third$\frac{1}{2}$ and repeat the question (answer 3/2. Continue.	egg carton fractions
Bring out models for one whole and find how many wholes are the same size as the pupil's halfs. Establish the concept of different names or labels for the same amount of number. (Also introduces pupil to the idea that we use fractions to talk about amounts greater than one whole.)	
(Note: sixths, twelfths, and eighths if using 30 egg flats.)	Change "4 colors to 6 or 7 colors".
Paper plate model: Another model for this activity is the round paper plate. If possible, buy a package of each color of several solid colors (cheaper plates are easier to cut). Use a protractor to measure the angles before cutting. The protractor degrees are:	

ACTIVITIES FOR LEARNING THE COMPETENCY MATERIALS

X. (b) cont. Handle and count fractional models (eg. egg cartons or fraction bars) to amounts larger than one whole (eg. 5/3).

1/2 = 180°
1/3 = 120°
1/4 = 90°
1/5 = 72°
1/6 = 60°
1/8 = 45°
1/9 = 40°
1/10 = 36°
1/12 = 30°

Use different colors for each fractional size. If colored plates are not available, use white plates and have children color all fractional parts of one size the same color.

This model is based on the popular "pie model" of fractions.